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A Full Symbolic Feedback Control Design

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ABSTRACT

Although Symbolic analysis suffers from higher complexity, higher resource requirements and longer execution than the numerical analysis, it is proven more accurate and general and it is recommended to use it. It is also proven that the feedback gains are very crucial to stabilize any system and it is important to measure them accurately. This paper introduces the design of a full symbolic feedback control system based on pole placement method. The feedback gains of parameter varying control system are estimated using three alternative algorithms; Direct substitution, Bass-Gura and Ackerman formula, where the gains can be changed according to the parameters which are measured online. Experiments were conducted on the aircraft pitch control as an application to estimate the feedback gain to stabilize the system. The results demonstrate that the symbolic solution reduces the complexity by a significant margin and produces the same results assumed in the compared research.

Keywords: Fully symbolic-based technique, pole placement techniques, Bass-Gura Method, Ackermann's formula, aircraft pitch control.

1. INTRODUCTION

Any system consists of a group of elements which are connected resulting in some parameters that affect the system. These parameters may change during the lifetime of any system due to time or event [1]. These changes range in intensity from slight like small disturbances that has almost no observed effect, to severe changes that can cause performance degradation or whole system failure [2-3].

Analysing this kind of systems is done using either numerical or symbolic methods. Numerical methods assume several sample points and the results are hold for all points between them. Despite of its spread, it may give wrong conclusions. In addition, some variables may be difficult to know their values or control which may introduce different values to be the best for that specific application. The symbolic-based method is a general method used to produce symbolic expressions for a system using its parameters and variables. It gives exact, general, and compact mathematical solution [4].

In [5] the symbolic technique is investigated and applied to different applications whose state space model is in small dimension. However, [6] proposed a fully symbolic-based technique which is used for solving general n^{th} order state-space model and then use the solution to analyse this parameter-variation system. In this technique, the state space model is solved using the partitioned matrix theory and block-wise inversion formula, so the process is speeded up and the computational complexity problem is reduced. In this research, a fully symbolic-based technique is applied to design a full state feedback control based on pole placement methods.

Every controller designer seeks the system to behave according to his demands. System behavior can be controlled through poles location. Figure 1 shows the relation between the poles' location and the system's behavior. The pole placement method allows to choose the poles according to your demands [7].

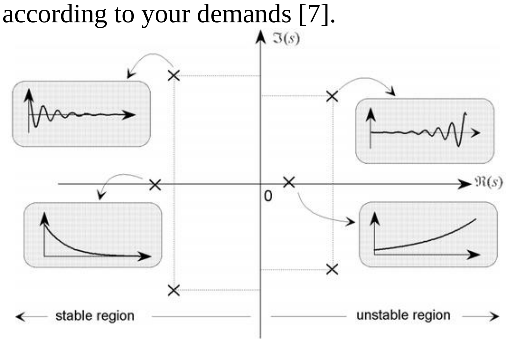

Figure 1: Relation between poles location and system behavior

In this paper, three methods of pole placement are applied to the control of the aircraft and compared the results with the numerical method and compared with the open loop control of it. The paper is organized as follows: The

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problem is formulated in Section 2. Pole placement methods are presented in Section 3. The discussion of the aircraft pitch control problem along with the simulation results and the comparison with the numerical technique are presented in Section 4. Finally, the paper is concluded in Section 5.

2. PROBLEM STATEMENT

The state space model of any system can be expressed as in group Eq. (1-2).

$$\dot{x} = A\,x + B\,u \tag{1}$$

$$y = C\,x + D\,u \tag{2}$$

where (1) is the state equation and (2) is the output equation. u is the input function and y is the output function. $A(n \times n)$ is the state matrix, $B(n \times p)$ is the input matrix, $C(q \times n)$ is the output matrix, and $D(q \times p)$ is the feed-through matrix. They can be expressed with symbolic parameters as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix} \tag{4}$$

$$C = [\, c_{11} \quad c_{12} \quad \cdots \quad c_{1n}] \tag{5}$$

Substitute u = - K x, we get,

$$\dot{X} = (\,A - B\,K\,)\,x \tag{6}$$

Obviously, the eigenvalues of (A-BK) matrix may specify the system's behaviour, where K represents the gain matrix. To calculate the gain matrix *K*, one of three standard different methods can be used: Direct substitution, Bass-Gura, and Ackerman formula. The necessary condition to apply the pole placement methods is to be sure that the system is state controllable. So the controllability matrix should be checked before either of the pole placement methods could be used.

3. POLE PLACEMENT TECHNIQUES

3.1. Direct Substitution Method

The steps that are followed to solve for gain K in the direct substitution method are shown in the pseudo code given below. This method should only be used for low order systems, as it becomes difficult when the state space dimensions grow higher (n > 3).

a. Algorithm: Direct Substitution Method Algorithm

INPUT: The state matrix $A(n \times n)$, The input matrix $B(n \times p)$ and the desired pole locations μ_1, .., μ_n.
OUTPUT: finding the gain vector $k = [\boldsymbol{k_1}, .., \boldsymbol{k_n}]$.
Begin
1- Determine the controllability matrix $\boldsymbol{CM}$,

$$\boldsymbol{CM} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A^2B} \ \cdots \ \boldsymbol{A^{n-1}B}]$$

2- Check the controllability condition, if *Rank (CM)* < *n*, report uncontrollable and stop.
Else go to Step 3.
3- Define the gain k vector as,

$$k = [\boldsymbol{k_1}, \boldsymbol{k_2}, \dots, \boldsymbol{k_n}]$$

4- Substitute the gain k in the desired characteristic polynomial equation,

$$|\mathbf{sI} - \mathbf{A} + \mathbf{B\,k}| = (s - \mu_1)\,..\,(s - \mu_n)$$

5- Equate the coefficients of the like powers of *s* on both sides.
6- Solve for $\boldsymbol{k_1}, \boldsymbol{k_2}, .., \boldsymbol{k_n}$.
End

3.2. Bass-Gura Method

Finding the gain matrix *K* using transformation matrix *T* is based on transforming the given system into controllable canonical form, then the poles are placed at the desired locations. It was found that Bass-Gura formula is efficient in most applications. The following pseudo code describes the steps that must be followed to solve for gain *K* using this method.

b. Algorithm: Bass-Gura Approach Algorithm

INPUT: The state matrix $A(n \times n)$, The input matrix $B(n \times p)$ and the desired pole locations μ_1, .., μ_n.
OUTPUT: finding the gain vector $k = [\boldsymbol{k_1}, .., \boldsymbol{k_n}]$.
Begin
1- Determine the controllability matrix $\boldsymbol{CM}$,

$$\boldsymbol{CM} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A^2B} \ \cdots \ \boldsymbol{A^{n-1}B}]$$

2- Check the controllability condition, if *Rank (CM)* < *n*, report uncontrollable and stop.
Else go to Step 3.
3- Form the characteristic polynomial for matrix A,

$$|\mathbf{SI} - \mathbf{A}| = \boldsymbol{s^n} + \boldsymbol{a_1}\,\boldsymbol{s^{n-1}} + \boldsymbol{a_2}\,\boldsymbol{s^{n-2}} + \dots + \boldsymbol{a_{n-1}}\,s + \boldsymbol{a_n}$$

Determine the values of coefficients $\boldsymbol{a's}$: $\boldsymbol{a_i} = [\,\boldsymbol{a_1}, \boldsymbol{a_2}, \dots, \boldsymbol{a_n}]$, $i = 1, \dots, n$.
4- Find the Transformation matrix *T*,

$$T = CM \times W$$

$$\text{where } W = \begin{bmatrix} \boldsymbol{a_{n-1}} & \boldsymbol{a_{n-2}} & \dots & \boldsymbol{a_1} & \mathbf{1} \\ \boldsymbol{a_{n-2}} & \boldsymbol{a_{n-3}} & \dots & \mathbf{1} & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots \\ \boldsymbol{a_1} & \mathbf{1} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

5- Write the desired characteristic polynomial using the desired closed-loop poles,

$(S - \mu_1)..(S - \mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_{n-1} s + \alpha_n$

Find the coefficients $\alpha's$: $\alpha_i = [\alpha_1, \alpha_2, .., \alpha_n]$, i = 1, …, n.

6- Find the required feedback gain matrix k using,

$K = [(\alpha_n - a_n)\ (\alpha_{n-1} - a_{n-1})\ ..\ (\alpha_1 - a_1)]T^{-1}$

End

3.3. Ackermann's formula

This method is based on Ackermann's algorithm (cited by Kailath and incorporated in the Matlab suite) which makes the gain computations easier and requires less work. It is an alternative to the Bass-Gura algorithm, and both yield the same results and suitable for larger systems. The following is the pseudo code that describes the steps that must be followed to solve for gain K in this method.

a. Algorithm: Ackermann's formula Algorithm

INPUT: The state matrix A(n×n), The input matrix B(n×p) and the desired pole locations $\mu_1, .., \mu_n$.

OUTPUT: finding the gain vector $k = [k_1, .., k_n]$.

Begin

1- Determine the controllability matrix CM

$$CM = [B \quad AB \quad A^2B \ \cdots \quad A^{n-1}B]$$

2- Check the controllability condition, if *Rank (CM) < n*, report uncontrollable and stop.

Else go to Step 3.

3- Write the desired characteristic polynomial using the needed closed-loop poles,

$(s - \mu_1)..(s - \mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_{n-1} s + \alpha_n$

Find the coefficients $\alpha's$: $\alpha_i = [\alpha_1, \alpha_2, .., \alpha_n]$, i = 1, …, n.

4- Form Obtain ϕ (A) where

$\phi(A) = A^n + \alpha_1 A^{n-1} + \ldots + \alpha_{n-1} A + \alpha_n I$

5- Find the required feedback gain matrix k using,

$K = [\,0\ 0\ 0\ \ldots\ 1]\ [\mathbf{B}\ \mathbf{AB}\ A^2 B\ \ldots A^{n-1} B\,]^{-1}\ \phi(A)$

End

The merits of performing exact calculations by using symbolic algebra in the state feedback controller were discussed in [8]. It was proved that when all calculations are carried out exactly (symbolically), the required state feedback controller gain (K) can be obtained without any numerical errors using any method of pole assignment, while using numerical methods results in a warning message indicating that the poles are greater than 10% in error. So, it was concluded that symbolic technique is better in the solution of numerically error prone problems such as pole assignment.

On other words, instead of applying the solution algorithm online in the execution stage using numeric techniques that takes quite some time, It is suggested to solve for the feedback gains formulas offline in a pre-processing stage using symbolic techniques that allow obtaining neat equations composed of basic mathematical operations that are calculated almost in no time. Generally, the computation time is quite long in the offline derivation stage. Nevertheless, this relatively long stage is only made once in return for much lower execution time in the online tracking stage.

4. EXPERIMENTS AND RESULTS

The aircraft control is discussed here, and the pole placement methods are used to calculate the feedback gain symbolically.

4.1. Aircraft pitch control

In aircraft designing, it is essential to evaluate and understand the performance, behavior, safety, and other features of the system before the operational testing. Through controlling ailerons, elevator and rudder, an aircraft has three translational paths: transverse, vertical, horizontal and three rotational paths: pitch, yaw, roll [9]. Under certain presumptions, the equations of motion of an aircraft can be separated then make it linear to longitudinal and transverse equivalent [10]. The main forces and coordinate axes applied to an aircraft are illustrated in Figure 2. A list defining all the used symbols for modelling aircraft pitch system is depicted in Table 1.

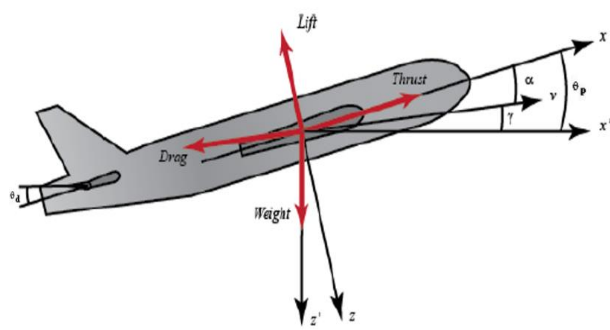


Figure 2: Aircraft selected state variables

Table 5.1: Aircraft model parameters definitions [11]

Parameter	Definition
α	Attack angle
q	Pitch rate
θ_p	Pitch angle
θ_d	Elevator deflection angle
C_D	Coefficient of drag
C_L	Coefficient of lift
C_W	Coefficient of weight
C_M	Coefficient of pitch moment
γ	Flight path angle

The aircraft is assumed to be in steady-cruise at constant speed and height, so the forces of thrust, drag, weight, and lift balance each other in both x- and y- directions. Hence, the aircraft's longitudinal motion equations can be written as follows [11],

$$\dot{\alpha} = \mu\Omega\sigma[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)} q - (C_W \sin\gamma)\theta_p + C_L]$$

$$\dot{q} = \frac{\mu\Omega}{2\, i_{yy}} [C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_W \sin\gamma)\theta_d$$

$$\dot{\theta}_p = \Omega q \quad (7)$$

In this case study, an autopilot controlling the pitch of the aircraft is investigated and the state space is used by taking three state variables into consideration which are the angle of attack denoted by α, pitch rate denoted by q, and pitch angle denoted by Θ_p. Here, Boeing commercial aircraft differential equation is employed [12]. The state-space representation used are,

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta_p} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta_p \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} \theta_d$$

$$y = [0 \;\; 0 \;\; 1] \begin{bmatrix} \alpha \\ q \\ \theta_p \end{bmatrix} \quad (8)$$

To apply our model, the state space model is re-written in symbolic form as follows,

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta_p} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & a_{32} & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta_p \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} \theta_d$$

$$y = [0 \;\; 0 \;\; 1] \begin{bmatrix} \alpha \\ q \\ \theta_p \end{bmatrix} \quad (9)$$

First, the necessary and sufficient condition to apply the pole placement methods is that the system must be completely state controllable [13-14]. The controllability formula (CM) can be calculated using,

$$CM = [\, A \mid AB \mid A^2B \mid A^3B \mid \dots \,] \quad (10)$$

MUPAD code was used to calculate the controllability matrix which yields,

CM=

$$\begin{bmatrix} b_1 & a_{11}b_1 + a_{12}b_2 & b_1({a_{11}}^2 + a_{12}a_{21}) + b_2(a_{11}a_{12} + a_{12}a_{22}) \\ b_2 & a_{21}b_1 + a_{22}b_2 & b_2({a_{22}}^2 + a_{12}a_{21}) + b_1(a_{11}a_{21} + a_{21}a_{22}) \\ 0 & a_{32}b_2 & a_{21}a_{32}b_1 + a_{22}a_{32}b_2 \end{bmatrix} \quad (11)$$

The rank of the controllability matrix is 3, so the system is state controllable. Now, any of the standard pole placement methods can be used to find the feedback gain formulas. In fact, the three methods were tested to check the efficiency and usage for each algorithm in its symbolic form. The comparative examination shows that the three methods give identical final numerical values, while the resulting symbolic equations vary in the simplification quality. Although not perfect, but the Bass-Gura method was the most simplified symbolic results compared to the other two methods, while the direct substitution method gives the worst final equation in terms of simplicity.

$$k_1 = \frac{b_2\,\sigma_3 - \sigma_4\,\sigma_5}{\sigma_2} - b_2\,\sigma_6$$

$$k_2 = \frac{-b_1\,\sigma_3 - \sigma_4\,(a_{12}b_2 - a_{22}\,b_1)}{\sigma_2} - b_1\,\sigma_6$$

$$k_3 = \frac{\mu_1\,\mu_2\,\mu_3}{a_{32}\sigma_5}$$

where,

$$\sigma_1 = b_1{b_2}^2\;({a_{11}}^2 - a_{11}a_{22} - a_{12}a_{21}) + a_{21}\,b_2\,{b_1}^2\,(a_{22} - 2\,a_{11}) + a_{11}\,a_{12}\,{b_2}^3 + {a_{21}}^2\,{b_1}^3$$

$$\sigma_2 = a_{12}{b_2}^2 - a_{21}{b_1}^2 + b_1\,b_2(a_{11} - a_{22})$$

$$\sigma_3 = a_{12}\,a_{21} - a_{11}\,a_{22} + \mu_3(\mu_1 + \mu_2) + \mu_1\,\mu_2$$

$$\sigma_4 = \mu_1 + \mu_2 + \mu_3 - a_{11} - a_{22}$$

$$\sigma_5 = a_{11}b_2 - a_{21}\,b_1$$

$$\sigma_6 = \frac{\mu_1\,\mu_2\,\mu_3 b_2}{\sigma_1} \quad (12)$$

4.2. Comparing between open loop and closed loop aircraft

By solving the aircraft model Eq. (8) using the symbolic-based technique introduced in [6], the solution for the aircraft model is derived as follows:

$$\alpha(t) = \alpha_0\sigma_5\,(\cosh\sigma_6\,t + \sigma_7) + a_{12}\,q_0\,\sigma_{16} + U\,(\frac{\sigma_2\sigma_{16}}{\sigma_4\sigma_6} + \frac{a_{12}b_2 - a_{22}b_1 + \sigma_5\,\sigma_{12}}{\sigma_4})$$

$$q(t) = q_0\sigma_5\,(\cosh\sigma_6\,t - \sigma_7) + \alpha_0\,a_{21}\sigma_{16} + U\,(\frac{\sigma_3\sigma_{16}}{\sigma_4\sigma_6} + \frac{a_{21}b_1 - a_{11}b_2 + \sigma_5\,\sigma_{11}}{\sigma_4})$$

$$\theta_p(t) = \theta_0 + \alpha_0(\frac{\sigma_{14}}{\sigma_4}) - q_0(\frac{\sigma_{15}}{\sigma_4}) + U(\frac{\sigma_{16}}{\sigma_1{\sigma_6}^3} + \frac{a_{32}t(a_{21}b_1 - a_{11}b_2)}{\sigma_4} + \frac{\sigma_{10}(1 - \sigma_5\cosh\sigma_6 t)}{\sigma_1})$$

where,

$$\sigma_1 = {a_{11}}^2\,{a_{22}}^2 - 2\,a_{11}\,a_{12}\,a_{21}\,a_{22} + {a_{12}}^2\,{a_{21}}^2$$

$$\sigma_2 = b_1({a_{11}}^2 - {a_{22}}^2) + a_{11}b_2(a_{11} + a_{22})$$

$$\sigma_3 = b_2({a_{22}}^2 - {a_{11}}^2) + a_{21}b_1(a_{11} + a_{22})$$

$$\sigma_4 = a_{11}\;a_{22} - a_{12}\,a_{21}$$

$$\sigma_5 = e^{t(\frac{a_{11}}{2} + \frac{a_{22}}{2})}$$

$$\sigma_6 = \sqrt{\frac{{a_{11}}^2}{4} - \frac{a_{11}a_{22}}{2} + \frac{{a_{22}}^2}{4} + a_{12}a_{21}}$$

$$\sigma_7 = \frac{\sinh(\sigma_6\, t)(\frac{a_{11}}{2} - \frac{a_{22}}{2})}{\sigma_6}$$

$$\sigma_8 = \frac{\sinh \sigma_6\, \mathrm{t}}{\sigma_6}\left(\frac{a_{11}+a_{22}}{2} - \frac{a_{11}a_{21}a_{32}+a_{22}a_{21}a_{32}}{a_{21}a_{32}}\right)$$

$$\sigma_9 = \frac{\sinh \sigma_6\, \mathrm{t}}{\sigma_6}\left(\frac{a_{11}+a_{22}}{2} - \frac{{a_{11}}^2 a_{32}+a_{22}a_{21}a_{32}}{a_{11}a_{32}}\right)$$

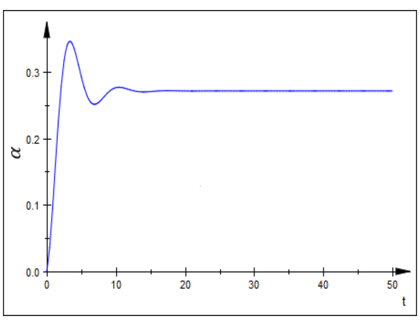
$$\sigma_{10} = b_1 a_{21}a_{32}(a_{11} + a_{22}) - a_{32}b_2\,({a_{11}}^2 + a_{12}a_{21})$$

$$\sigma_{11} = -b_2\sigma_6 \sinh \sigma_6\, t + \cosh \sigma_6\, t(a_{11}b_2 - a_{21}b_1)\sigma_{12} = -b_1\sigma_6 \sinh \sigma_6\, t + \cosh \sigma_6\, t(a_{22}b_1 - a_{12}b_2)\sigma_{13} = a_{32}({a_{11}}^2 a_{21}b_1 - {a_{11}}^3 b_1 + {a_{22}}^2 a_{11}b_2)\sigma_{14} = a_{21}a_{32} - a_{21}a_{32}\sigma_5\,(\cosh \sigma_6\, t + \sigma_8)$$

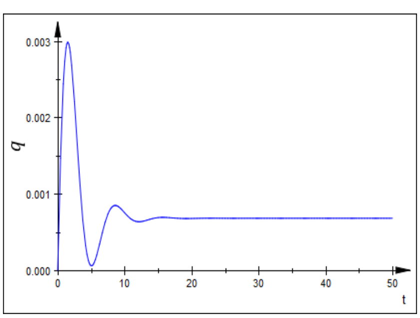
$$\sigma_{15} = a_{11}a_{32} - a_{32}a_{11}\sigma_5\,(\cosh \sigma_6\, t + \sigma_9)$$

$$\sigma_{16} = \frac{\sigma_5 \sinh \sigma_6\, t}{\sigma_6} \qquad (13)$$

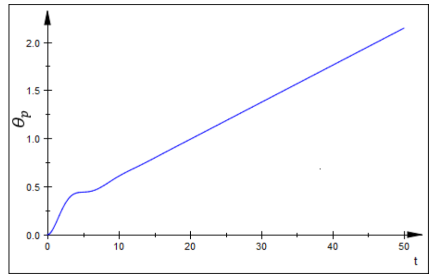
Real aircraft parameters are used for testing to have and discuss simulation results [12]. Figure 3 shows the open-loop system response. From the plot given below, it is apparent the instability of the pitch angle response. Thus, the open-loop response does not satisfy the design criteria at all.



(a) α versus time



(b) q versus time



(c) θ versus time

Figure 3: The open-loop system response

In order to stabilize the system, the feedback controller is added with gain matrix *K* given in Eq. (12) and the pitch angle response is re-plotted. Examining Figure 4 which depicts the closed loop response for the pitch angle, feedback gain must be added to the system to make it stable.

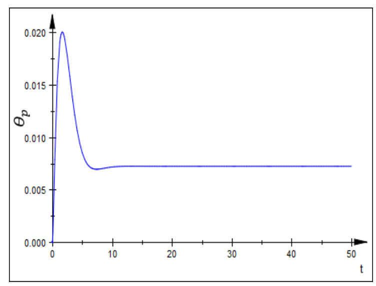


Figure 4: The closed-loop system response.

4.3. Comparing with numerical method

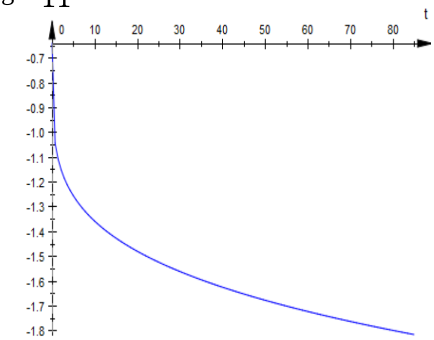
For further verification of the symbolic output, the MATLAB numerical function *acker* is used. It works for single-input single-output (SISO) systems only with a small number of state variables [15]. It requires besides A and B matrices, a vector of the wanted closed-loop pole locations. So, the dominant poles approximation approach is used. It considers that the response is dominated by the slowest part of the system, while the faster parts can be neglected. Based on this assumption, a pair of conjugate poles which represent the required gains such as overshoot and settling time is chosen and positioned, then the higher-order poles is placed about ten times farther than the dominant poles to the left of the complex plane.

The equations describing the approximation is given in [16]. The design criteria are chosen to have overshoot less than 5% and settling time less than 10 seconds. The resulted outputs show identical values to that generated from the symbolic equations. The numeric gain output after running the program,

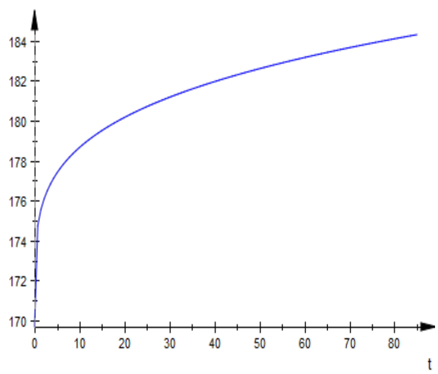
K = [-20.0487   585.2367   27.6823]

4.4. Parameter variations Analysis

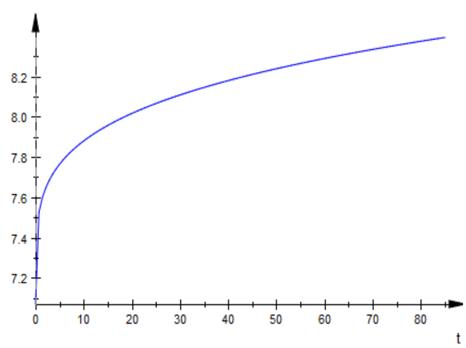
By studying the parameter variation effect on the feedback gains which calculated in Equation (12), one parameter (a_{ij}) must be changed at a time while the other parameters are kept unchanged. Then draw the parameter varying analysis curves. Figure 5 shows the effect of changing a_{11} with time.



(a) k_1 versus time.



(b) k_2 versus time.



(c) k_3 versus time.

Figure 5: The effect of changing a_{11} on the gain values.

Experimental results show that the symbolic solution gives the same results as the numerical solution when calculating the feedback gains to keep the stability of the system. Hence, the symbolic based model can be a good alternative to the numerical method, and it has the upper hand of generality and flexibility. Also, the symbolic functions tracked parameter changes successfully with no possibility to error prone unlike the numerical methods.

The motivation of our work also is to reduce the complexity by attaining final neat equations that do not have high space and time complexity. The complexity of applying the fully symbolic-based technique is O(1) which describes that the algorithm always takes constant execution time (or space) regardless of the size of the state-space matrices and applies only simple arithmetic operations such that addition and multiplication on elements. On the other hand, the complexity of using the numeric technique in pole placement algorithms is $O(n^3)$, which means that the length of time the algorithm takes to execute will grow linearly and in direct proportion to the size of the state-space matrices and applies the arithmetic operations on the whole matrices.

5. CONCLUSION

A fully symbolic based technique was introduced for the derivation of feedback controller gains. The goal of the new technique is to have neat equations that are applied easily. Simulation results reveal that the proposed method has flexibility and generality that gives a good picture of the system being investigated. Moreover, it provides the same results if compared to the numerical model. Also, it reveals a non-error prone methodology with successful tracking and adapting to the parameters variations. Later, the fully symbolic method will be applied to different types of systems and controllers.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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