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Sensitivity Analysis of XY- Precision Positioning Compliant Mechanism

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ABSTRACT

Compliant mechanism becomes a promising alternative to the traditional ones for precision applications. Flexure hinge compliant mechanisms have been used in different fields. However, the design of their geometric parameters is so critical. Therefore, a lot of work are done in optimization of their design parameters. One of the main important steps is to select the sensitive parameters in such design problems. In this article a study of sensitivity of geometric parameters of positioning stage compliant mechanism are represented to be used as a pilot step for the selection of the optimization design variables. The effect of each parameter is regarding to the achieved amplification ratio of the positioning stage and the input stiffness of the mechanism. The modeling is done using Pseudo-Rigid-Body Model on MATLAB. The sensitivity analysis is done using ISIGTH software. The results of different sensitivity analysis methods reveal the most important parameters of the compliant mechanism that have significant effect on amplification ratio.

Keywords: Compliant Mechanism, Precision Positioning, Sensitivity Analysis, Pseudo Model.

1. INTRODUCTION

Compliant mechanisms play important role in precision engineering recently. Flexure hinge compliant mechanisms have been used in different fields such as micro-nano positioning, micro-gripping, metrology instruments, bio-medical devices, and MEMS. The flexibility of the mechanism is responsible for smooth and precise motions. The monolithic structure provides high precision, no assembly, no lubrication, less expensive to manufacture and maintain than conventional devices [1,2].

The optimization of the geometric parameters of such mechanisms are one of the important design procedures to control their performance. The optimization can be simplified, by focusing on the significant parameters. These parameters are selected based on sensitivity analysis. The objective of this paper is to study the sensitivity of the geometric parameters of the investigated positioning-stage compliant mechanism. The sensitivity analysis is considering the objective function to be the amplification ratio and input stiffness. The analysis is based on using Pseudo-Rigid-Body Model (PRBM) [3]. The PRBM is modeled using MATLAB and integrated with ISIGHT software to get the influence of input parameters on the output variable with different methods of design of experiments.

The paper organization is as following: section 2 deal with PRBM and the equations used to model the investigated mechanism in MATLAB. Sensitivity analysis is performed in section 3 with different methods. Section 4 shows the results of the sensitivity analysis. Comments on the results and conclusion in section 5.

2. PSEUDO-RIGID-BODY MODEL

As to the kinematic and static analysis of compliant mechanisms, many theoretical methods are now available such as the PRBM, the matrix method, the elastic beam theory and Castigliano's second theorem [2,3].

The PRBM approach replaces the traditional kinematic joint with a suitable compliant joint with lumped or distributed compliance [4]. Using the pseudo rigid body model approach, compliant mechanism is modeled as rigid links connected by pin joints and torsional springs [5,6].

The amplification of the piezo electric actuator motion in the investigated positioning stage compliant mechanism [7] comes from using combination of rhombus-type and lever-type as shown in Figure 1. The mechanism consists of flexure beams and flexure hinges, each one has two nodes with three degree of freedom per node. The hinges are elliptical type. The dynamic stiffness matrix of each element is used to get frequency-dependent relationship between the nodal force (F^e) and the nodal displacement (x^e), this can be expressed as following [8].

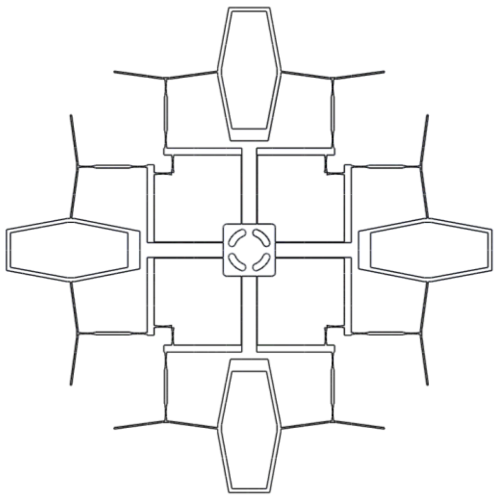


Figure 1: Positioning stage compliant mechanism

$$F^e = D^e . x^e \tag{1}$$

$$D^e = \begin{bmatrix} d_1 & 0 & 0 & d_5 & 0 & 0 \\ & d_2 & -d_3 & 0 & d_6 & d_7 \\ & & d_4 & 0 & -d_7 & d_8 \\ & & & d_1 & 0 & 0 \\ & sym & & & d_2 & d_3 \\ & & & & & d_4 \end{bmatrix} \tag{2}$$

where d_i; are the coefficients of the dynamic stiffness matrix D^e.

2.1. Beam

For beam, the nodal force (F^e) and the nodal displacement (x^e) of a flexible beam can be expressed using the dynamic stiffness matrix (D^e) [5]. The Euler-Bernoulli beam theory is used to get the dynamic stiffness matrix of flexible straight beam [9].

Equations from (3) to (12) describing the coefficients of the dynamic stiffness, where E is modulus of elasticity of the material, A is the cross-section area of beam, l is the length of beam, α and β are two factors, ρ is the density of beam material, ω is the frequency.

$$d_1 = \frac{EA\alpha \cot\alpha}{l} \tag{3}$$

$$d_2 = \frac{EI\beta^3(\cos\beta \sinh\beta + \sin\beta \cosh\beta)}{l^3(1 - \cos\beta \cosh\beta)} \tag{4}$$

$$d_3 = -\frac{EI\beta^2(\sin\beta \sinh\beta)}{l^2(1 - \cos\beta \cosh\beta)} \tag{5}$$

$$d_4 = \frac{EI\beta(\sin\beta \cosh\beta - \cos\beta \sinh\beta)}{l(1 - \cos\beta \cosh\beta)} \tag{6}$$

$$d_5 = -\frac{EA\alpha \csc\alpha}{l} \tag{7}$$

$$d_6 = -\frac{EI\beta^3(\sin\beta + \sinh\beta)}{l^3(1 - \cos\beta \cosh\beta)} \tag{8}$$

$$d_7 = \frac{EI\beta^2(\cosh\beta - \cos\beta)}{l^2(1 - \cos\beta \cosh\beta)} \tag{9}$$

$$d_8 = \frac{EI\beta(\sinh\beta - \sin\beta)}{l(1 - \cos\beta \cosh\beta)} \tag{10}$$

$$\alpha^2 = \frac{\omega^2 l^2 \rho}{E} \tag{11}$$

$$\beta^4 = \frac{\omega^2 l^4 \rho A}{EI} \tag{12}$$

By simplifying the coefficients using power series, the dynamic stiffness matrix of the beam (D^b) can be written as following:

$$D^b = K_o - \omega^2 M_1 - \omega^4 M_2 - \omega^6 M_3 - \cdots \tag{13}$$

M1, M2 …. are terms of dynamic stiffness matrix. In case of low frequency, the high order terms are very small so can be ignored with reasonable approximation and the dynamic stiffness matrix of the beam (D^b) in this case separate to elastic stiffness matrix and mass matrix as the traditional model as shown in the equations (14), (15) and (16).

$$D^b = K_o - \omega^2 M_1 \tag{14}$$

$$K_O = \frac{E}{l^3}\begin{bmatrix} Al^2 & 0 & 0 & -Al^2 & 0 & 0 \\ & 12I & 6Il & 0 & -12I & 6Il \\ & & 4Il^2 & 0 & -6Il & 2Il^2 \\ & & & Al^2 & 0 & 0 \\ & sym & & & 12I & -6Il \\ & & & & & 4Il^2 \end{bmatrix} \tag{15}$$

$$M_1 = \frac{\rho Al}{420}\begin{bmatrix} 120 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22l & 0 & 54 & -13l \\ & & 4l^2 & 0 & 13l & -3l^2 \\ & & & 140 & 0 & 0 \\ & sym & & & 156 & -22l \\ & & & & & 4l^2 \end{bmatrix} \tag{16}$$

2.2. Lumped mass

For lumped mass, the platform which gets the output motion due to the piezo-electric actuator is simplified as a lumped mass and the dynamic stiffness matrix of a lumped mass, according to Newton second law, is expressed as following [8]:

$$M_n(\omega) = \begin{bmatrix} -m\omega^2 & 0 & 0 \\ 0 & -m\omega^2 & 0 \\ 0 & 0 & -J\omega^2 \end{bmatrix} \tag{17}$$

2.3. Flexure hinge

For elliptical flexure hinge used in this compliant mechanism, it is complicated to determine the exact dynamic stiffness matrix because of the variation of cross section of flexure hinge, so the flexure hinge is represented as massless spring with two lumped masses at its ends, then the dynamic stiffness matrix equals the summation of the elastic stiffness matrix (K^h) and the mass matrix [8].

$$D^h = K^h - \omega^2 M \tag{18}$$

$$K_i^{(h)} = \begin{bmatrix} k_x & 0 & 0 & -k_x & 0 & 0 \\ & k_y & -k_\alpha & 0 & -k_y & -k_\alpha \\ & & k_\theta & 0 & k_\alpha & -l.k_\alpha - k_\theta \\ & & & k_x & 0 & 0 \\ & sym & & & k_y & k_\alpha \\ & & & & & k_\theta \end{bmatrix} \tag{19}$$

$$k_x = \frac{1}{c_x} \tag{20}$$

$$k_y = \frac{c_\theta}{c_\theta c_y - c_\alpha^2} \tag{21}$$

$$k_\theta = \frac{c_y}{c_\theta c_y - c_\alpha^2} \tag{22}$$

$$k_\alpha = \frac{-c_\alpha}{c_\theta c_y - c_\alpha^2} \tag{23}$$

$$c_x = \frac{a}{Edb} N_1 \tag{24}$$

$$c_y = \frac{ka}{Gdb} N_1 + \frac{24\,a^3}{Edb^3} N_2 - \frac{12\,a^3}{Edb^3} N_4 \tag{25}$$

$$c_\alpha = \frac{12a^2}{Edb^3} N_2 \tag{26}$$

$$c_\theta = \frac{12a}{Edb^3} N_2 \tag{27}$$

$$N_1 = \frac{2(2s+1)}{\sqrt{4s+1}} \arctan\left(\sqrt{4s+1}\right) - \frac{\pi}{2} \tag{28}$$

$$N_2 = \frac{12s^4(2s+1)}{(4s+1)^{5/2}} \arctan\left(\sqrt{4s+1}\right) + \frac{2s^3(2s+1)(6s^2+4s+1)}{(2s+1)^2\,(4s+1)^2} \tag{29}$$

$$N_4 = \frac{48s^5+8s^4+20s^3+30s^2+10s+1}{2\,(4s+1)^{5/2}} \arctan\left(\sqrt{4s+1}\right) - \frac{s(2s+1)^2\,(-12s^3+2s^2+6s+1)}{(2s+1)^2\,(4s+1)^2} - \frac{\pi}{8} \tag{30}$$

According to Ling [7], the material selected was Aluminum 7A04 which had Young's modulus of 71GPa, Poisson's ratio of 0.33 and density of 2770 Kg/m^3. The dimensions piezo stacks used in the modeling are 10mm*10mm*36mm and the axial stiffness (k_{pzt}) of 100 N/μm. The overall dimensions of the mechanism were 160mm*160mm*10mm and geometric parameters dimensions are shown in Figure 2. The shape of elliptical flexure hinge and the parameter of it is shown in Figure 3. The mean values of geometric parameters dimensions are stated in Table 1. The amplification ratio is the ratio between the platform motion to the motion of piezo electric actuator in the same direction.

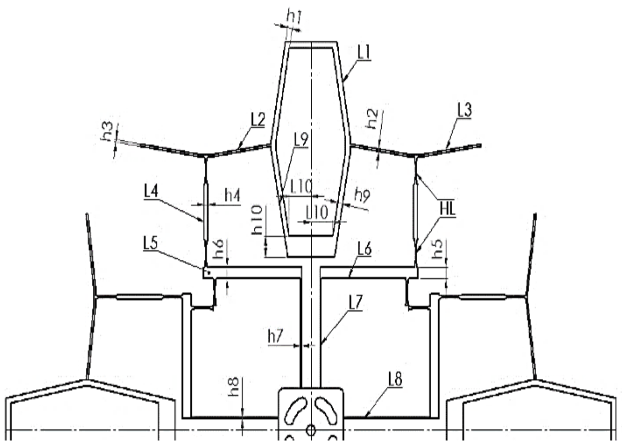


Figure 2: Schematic drawing of investigated positioning stage compliant mechanism

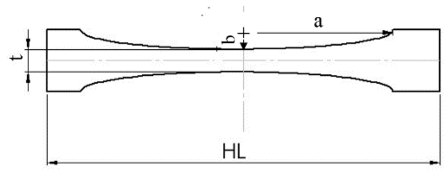


Figure 3: Schematic drawing of flexure hinge

Table 1. The mean values of geometric parameters

Parameter	[mm]	Parameter	[mm]
L1	16.7	h1	0.4
L2	14.1	h2	0.4
L3	14.1	h3	0.4
L4	10.0	h4	0.8
L5	2.0	h5	2.0
L6	20.0	h6	2.0
L7	20.0	h7	0.16
L8	20.0	h8	0.16
L9	16.7	h9	1.1
L10	5.0	h10	2.6
a	2.5	HL	5.2
b	0.3	t	0.16

3. SENSITIVITY ANALYSIS

Sensitivity analyses are applied using different methods and techniques such as parameter study method, full factorial method, fractional factorial method, Latin hypercube method, optimal Latin hypercube method and orthogonal array method and so on [10,11].

The effect of each parameter of the compliant mechanism is studied using three techniques: the parameter study method, the optimal Latin hypercube method and the orthogonal array method. Figure 4 shows the methodology of each technique. The sensitivity analysis is executed using ISIGHT software to get the influence of each parameter on the amplification ratio and the input stiffness.

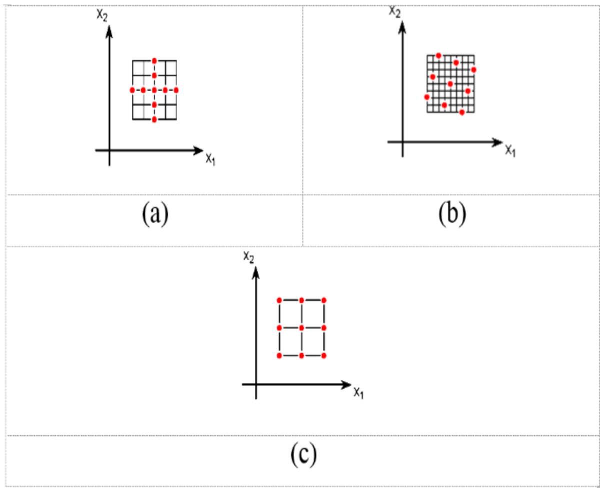


Figure 4: DOE methods: a) Parameter study method, b) Optimal Latin hypercube method and c) Orthogonal array method

4. RESULTS AND DISCUSSION

To study the influence of design variables on the displacement amplification ratio and input stiffness of the mechanism, the ISIGHT software was used. The design of experiment was carried out using three methods based on ISIGHT software integrated with MATLAB. parameter study method, optimal Latin hypercube method and orthogonal array method. All dimensions of the compliant mechanism had been studied as design variables to show the effect of each parameter on both the amplification ratio and the input stiffness.

Figure 5 shows the influence of each design variable using the parameter study method. The red color refers to negative effect and blue refers to the positive.

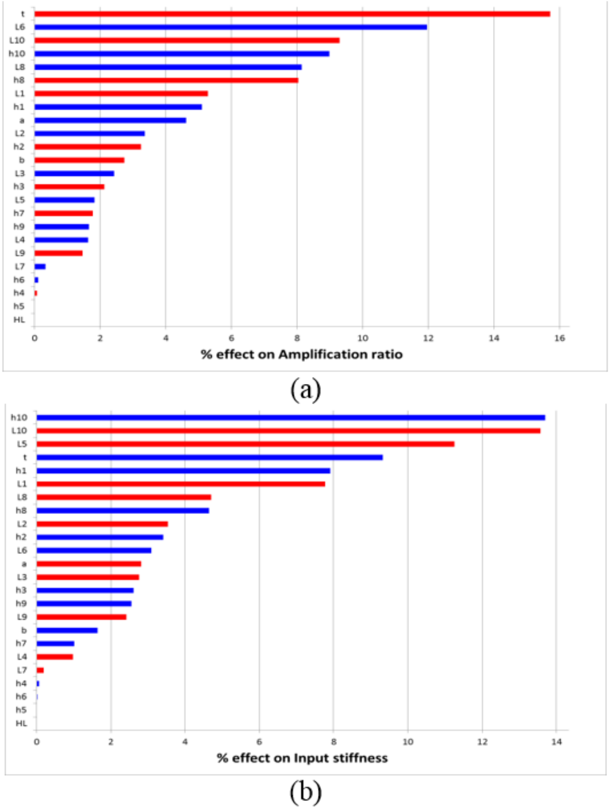


Figure 5: The effect of parameters using parameter study method on: a) Amplification ratio and b) Input stiffness

The results of the optimal Latin hypercube method and the orthogonal array method are shown in Figure 6 and Figure 7 respectively.

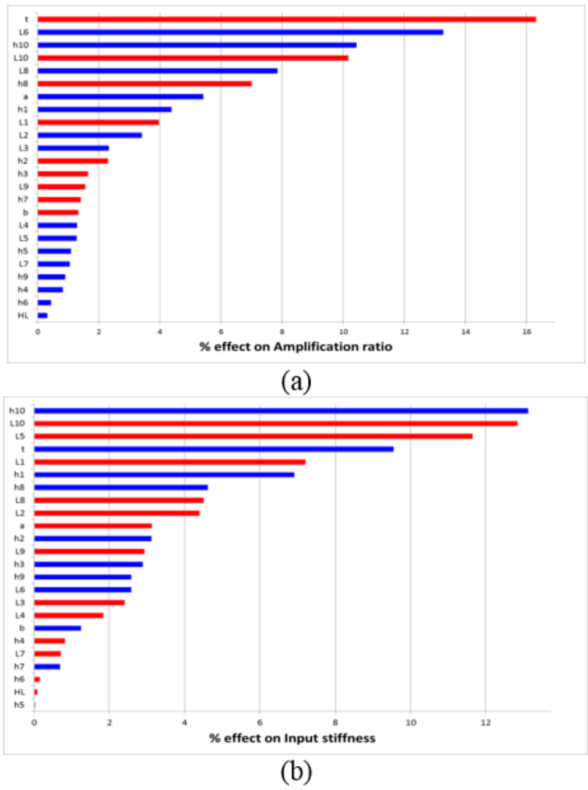
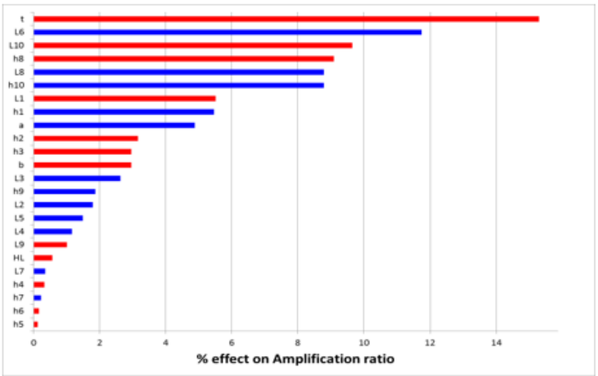
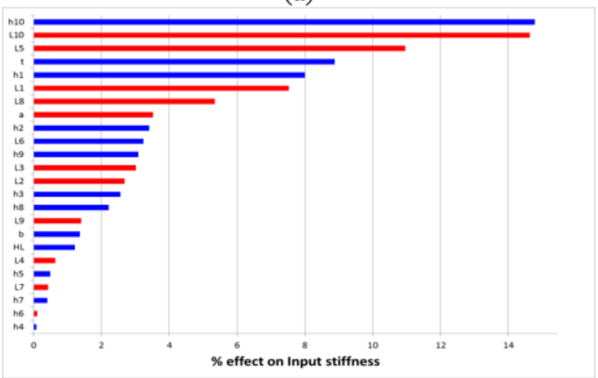


Figure 6: The effect of parameters using optimal Latin hypercube method on: a) Amplification ratio and b) Input stiffness



(a)



(b)

Figure 7: The effect of parameters using orthogonal array method on: a) Amplification ratio and b) Input stiffness

From the result graphs of each method, it can be noticed the most influence parameters on whether the amplification ratio or the input stiffness. The results are shown in Table 2.

Table 2. The DOE results summary

The DOE method	Amplification ratio		Input stiffness	
	Parameter	*Effect%*	*Parameter*	*Effect%*
The parameter study method	t	-15.72	h10	13.69
	L6	11.96	L10	-13.56
	L10	-9.30	L5	-11.24
The optimal Latin hypercube	t	-16.30	h10	13.13
	L6	13.26	L10	-12.85
	h10	10.43	L5	-11.65
The orthogonal array method	t	-15.30	h10	14.78
	L6	11.74	L10	-14.63
	L10	-9.64	L5	-10.97

5. CONCLUSIONS

For the amplification ratio, the DOE methods used, the parameter study method, the optimal Latin hypercube method and orthogonal array method, show that the flexure hinge thickness (t) has the great influence in the amplification.

The thickness of rhombus base (h10) plays significant role in the input stiffness of the compliant mechanism according to three DOE methods, the parameter study method, the optimal Latin hypercube method, and orthogonal array method.

The geometry of the flexure hinge (a, b and t) and the rhombus-type had significant effect on both amplification ratio and input stiffness.

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