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Numerical Analysis of Functionally Graded Material (FGM) Axisymmetric Cylinder under Transient Thermal Load and Variable Internal Pressure

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ABSTRACT

Metal-Ceramic Functionally Graded Materials (FGM) find extensive applications in nuclear power generators, spacecraft systems, and energy conversion systems. This study aims to investigate the behavior of FGMs under transient thermal loads and time-dependent mechanical loads through finite element modeling. A 2D model is developed using COMSOL Multiphysics to simulate the behavior of a metal-ceramic FGM thick cylinder. The model considers Titanium-Zirconium-Molybdenum (TZM) alloy for metals and Silicon carbide (SiC) for ceramic. It accounts for the effects of transient thermal loads and internal time-dependent pressure exerted on the cylinder's inner surface. Thermo-mechanical properties, assumed to follow an exponential function, are incorporated into the model. Numerical analysis results are compared with established analytical solutions to validate the model. Graphical comparisons between numerical and analytical results demonstrate significant agreement, confirming the reliability of the numerical approach. This study provides insights into the behavior of metal-ceramic FGMs under transient thermal and mechanical loads. The findings underscore the effectiveness of finite element modeling in understanding complex interactions within FGMs. Future research directions include exploring more complex geometrical models, incorporating additional loading scenarios, and performing experimental validation to enhance understanding of FGM performance in real-world conditions.

Keywords: FGM cylinder, Exponential properties, Transient thermal load, Time dependent pressure, COMSOL.

1.INTRODUCTION

A remarkable development in the research field concerning the functionally graded material (FGM) has been achieved in the last couple of decades. FGMs are those materials designed in such a way that their properties are changing relative to its dimensions in one or more defined directions. And it may categorize as; chemical composition gradient functionally gradient materials, porosity gradient functionally gradient materials, and microstructure gradient functionally gradient materials [1–3]. As the chemical composition gradient functionally are obtained by grading the chemical composition of two different distinct materials, porosity gradient functionally gradient materials are formed though gradually distributing the porosities through the material, and microstructure gradient functionally gradient materials are designed through changing the chemical composition within the same material which usually achieved through solidification process.

The earliest introduction of FGMs dates back to the 1980s when Japanese researchers aimed to enhance the properties of materials used in aerospace components. The FGM considered as the new generation of the composite materials, rapidly attracted the attention of researchers to study. Therefore, the studies extended to different application field beside the aerospace such as marine, nuclear and military industries, energy sector, medical field, communication field, automotive industries, etc. [4–8].

This study concerns the chemical composition gradient functionally gradient materials type. For this type, the two materials constituents are changed gradually and proportionally in the preidentified direction based on grading indices. Hence those indices have the greatest effect on the FGM properties [9–12]. Some scholars explored the grading of material properties based on power law functions [13, 14], while others were concerned on studying the sigmoid function [15], and others were interested in studying the exponential grading of the properties [8, 9, 16–18].

FGMs are designed to withstand harsh working environments, especially those include thermal loads or require high thermal resistance properties [19–23]. Hence, studying the FGM under different thermal loads has great interest. Das et al. [12] investigated a steady state heat transfer through a circular sandwich structure with convective radiative boundary condition. An analytical and finite element (FEA) solution are obtained and the findings are compared. Sharma et al. [24] studied FGM hollow cylinder considering thermal load as two different temperature profiles along with different inhomogeneity parameters. The variation of material properties is modeled using a power law function. Numerical solutions using finite element method (FEM) are found, concluding that stresses and strains within the cylinder are significantly depend on both the temperature profile for which the cylinder is subjected and the inhomogeneity parameters.

Additional studies have examined the effects of mechanical loads in conjunction with thermal loads. For instance, Gharooni et al. [25] employed the third-order shear deformation theory (TSDT) to derive an analytical solution to deduce the stresses and deformations of an FGM cylindrical shell under internal pressure along with thermal loading. The results compared with those obtained from FEM. Benslimanea et al. [26] provided both analytical and numerical approaches for calculating mechanical stresses and displacements in a thick-walled cylinder composed of FGMs subjected to thermal and mechanical loads. The thermal and mechanical properties are hypothesized to change radially based on a power law function, with a constant Poisson's ratio assumed. An exact solution of the displacement and stress distributions is obtained and validated using a numerical solution found using finite element method. Bayat et al. [27] applied a thermo- mechanical analysis of FGM sphere for different cases of pressure, temperature, and combined loadings separately. Moreover, different material properties grading indices values are considered and the results were compared to those obtained by FEM simulations. Bezzie et al. [28]. presented the influence of material grading index on the elastic responses of thick-walled axi-symmetrical pressure vessel experiencing uniform heat and pressure on both the inner and outer surface under plain strain conditions. Considering the thermo-mechanical material properties to vary gradually along the cylinder thickness direction, the analytical solution for the ceramic-metal FGM cylinder is obtained using Jupiter, Python open-access software, and subsequently graphically reported. The results emphasized the influence of the grading index on the obtained profile of stresses and strains within the FGM cylinder.

Due to the significance and crucial nature of the transient conditions in different working environments, Studying the FGM under various transient loading conditions has been an interest of researchers. Mousavi et al [29] developed an analytical method to address transient mechanical and thermal stresses in a thick FGM cylinder with integrated piezoelectric layers. Assuming the thermo-mechanical properties to vary in the radial direction according to power functions, a direct approach is employed for the solution of the Navier's equations, utilizing the complex Fourier series and the Euler equation. Verma et al. [30] considered a mechanical and thermal FGM cylindrical and spherical shell of revolution subjected to rapidly thermal and mechanical loads on the inner surfaces. Numerical results obtained using MATLAB for the nonlinear 1D heat conduction equation based on higher-order shear deformation theory for shells and Newmark time integration scheme.

Various software applications are employed for the validation of analytical results of stresses and strains on FGMs. In many cases, identifying the FGMs remains challenging as the material properties are following certain functions. Hence the significance of employing COMSOL Multiphysics, a commercial software, for simulation and analysis of the FGMs is justified. Bhardwaj et al. [31] investigated the steady state elastic stresses in an axisymmetric thick FG cylinder under axial temperature change using an iteration technique and FEM simulation using COMSOL Multiphysics. Sigmoid function is employed for the material properties gradation. The obtained analytical and FEM results are compared and showed good agreement. Paul et al. [32] investigated the steady state elastic stresses in a thick axisymmetric FG cylinder utilizing an iteration technique and FEM based solver COMSOL Multiphysics. Considering a FG cylinder with exponentially varying material properties and two material constituents of High Carbon Steel (HCS) as metal and Magnesium Oxide (MgO) as ceramic, the iterative and numerical simulation solution are found and compared graphically showing good agreement. Hoang et al [33] proposed a mesh free Radial Point Interpolation Method (RPIM) in order to solve a coupled thermo-mechanical FGM problem and compared the results with the FEM results achieved by COMSOL Multiphysics. In many cases, identifying the FGMs remains challenging as the material properties are following certain functions. Hence the significance of employing COMSOL Multiphysics, a commercial software, for simulation and analysis of the FGMs is justified. Multiphysics to verify the reliability of this method. Fu et al. [34] demonstrated the effectiveness of a semi-analytical boundary collection method for deploying a heat conduction analysis of FGM by comparing the results with COMSOL Multiphysics simulation results.

The novelty of this study lies in the comprehensive investigation of FGM pressure vessel behavior under simultaneous transient thermal and time-dependent mechanical boundary conditions, a research aspect often overlooked in existing literature. Advanced numerical analysis techniques, particularly utilizing COMSOL Multiphysics, are employed to elucidate the complex interplay between these boundary conditions and their effects on FGM pressure vessel performance. This integrated approach addresses a significant gap in the literature, which has traditionally focused on examining either transient thermal or time-dependent mechanical boundary conditions in isolation. Additionally, it serves the unique purpose of validating the outcomes reported in reference [35] through meticulous consideration of identical material constituents and boundary conditions, facilitating a comprehensive graphical comparison of results. Through this study, valuable insights into the behavior of FGM pressure vessels under realistic operating conditions are provided, contributing to the advancement of knowledge in this field.

2.NUMERICAL ANALYSIS

In this study, a rigorous numerical approach is employed using COMSOL Multiphysics to analyze the thermo-mechanical behavior of a metal-ceramic functionally graded material (FGM) thick cylinder subjected to transient thermal loads and internal pressure. To ensure transparency and reproducibility, a structured methodology encompassing key steps from problem definition to result validation is outlined. The following flowchart (Fig. (1)) illustrates the systematic process undertaken in our numerical analysis, guiding through each stage with clarity and precision.

The flowchart outlines the comprehensive numerical analysis process used in the study. This process includes the following key steps:

Initiate Study: The study begins with defining the problem and setting up the initial parameters.

Problem Definition: This involves specifying the geometry, material properties, and boundary conditions of the FGM cylinder.

Model Setup: The model is configured in COMSOL Multiphysics, including the selection of appropriate physics modules and coupling mechanisms.

Mesh Generation: A detailed mesh sensitivity analysis is conducted to determine the optimal mesh density. Quadrilateral elements are chosen for their computational efficiency and suitability for the geometry.

Solver Selection: The appropriate solver settings are configured for time-dependent analysis, considering the dynamic nature of the internal pressure and thermal loads.

Numerical Simulation: The simulations are run to solve the defined problem using the selected mesh and solver settings.

Results Analysis: The results are analyzed to evaluate the stress distributions and other relevant parameters.

Sensitivity Analysis: This step involves assessing the impact of varying parameters on the simulation results to ensure robustness.

Validation: The results are validated against theoretical or experimental data to ensure accuracy.

Mesh Convergence: A convergence test is performed to confirm that the mesh density is optimal and further refinement does not significantly alter the results.

Reporting: The validated results are compiled and documented for reporting.

End Study: The study is concluded after all analyses and validations are complete.

This structured approach ensures the reliability and reproducibility of the current numerical analysis, providing a robust framework for evaluating the performance of FGMs under transient thermal and variable internal pressure conditions.

Presented below is an in-depth exposition of the numerical analysis approach employed within this study. Aligned with the structured methodology delineated in the preceding flowchart (Fig. (1)), this explanation provides insight into the various stages of the numerical analysis process undertaken:

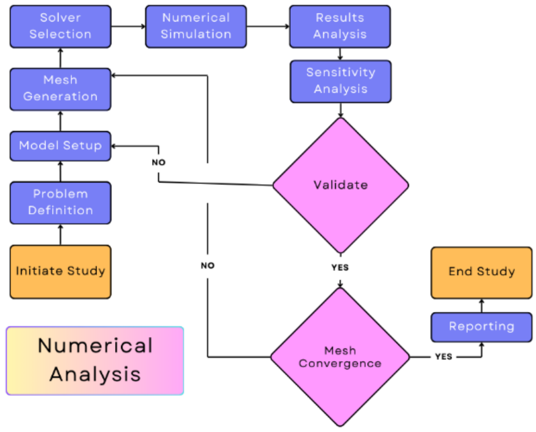

Figure 1: Flowchart illustrating the structured methodology for numerical analysis of metal-ceramic functionally graded material (FGM) thick cylinder behavior under transient thermal loads and internal pressure.

2.1. Problem Definition

Consider an axisymmetric thick cylinder constructed from ceramic-metal FGM, as illustrated in Fig. (2). This cylinder is characterized by an inner radius (R_i = 0.33m), an outer radius (R_o= 0.4m), and a finite length (L=1.0m). It undergoes dynamic variations in internal pressure (P) over time (t) and is subjected to transient radial temperature fluctuations (T). The FGM cylinder's interior and exterior walls are constructed using ceramic, specifically "Silicon carbide" (SiC), and metal, employing the molybdenum alloy "titanium-zirconium-molybdenum" (TZM), respectively.

The radial distribution of the thermos-mechanical material characteristics across the thick-walled FGM cylinder is presented by Habib et. al. [35] as follows:

$$\begin{aligned} E &= E_o\, exp\hat{}(\beta(1 - r/R_o)) \\ \alpha &= \alpha_o\, exp\hat{}(\varphi(1 - r/R_o)) \\ \lambda &= \lambda_o\, exp\hat{}((\gamma(1 - r/R_o)) \\ \rho &= \rho_o\, exp\hat{}(\psi(1 - r/R_o)) \\ C_p &= C_{p_o}\, exp\hat{}(\zeta(1 - r/R_o)) \end{aligned} \qquad (1)$$

Where, E and ρ represents the mechanical properties of the FGM, specifically Young's modulus and density, respectively, which vary as functions of the radial distance r. The parameters β, and ψ denote the respective material indices. Similarly, α, λ and Cp signify the thermal properties of the FGM, namely thermal expansion coefficient, thermal conductivity coefficient, and specific heat capacity, which also varying with r. The symbols ϕ , γ, and ζ serves as the respective material grading indices, with the assumption of a constant Poisson's ratio. Furthermore, the symbols E_o, and ρ_o stands for the mechanical properties of the FGM cylinder at its outer radius Ro, representing Young's modulus, and density, respectively. Similarly, α_o, γ_o, and C_{p_o} represent the thermal properties of the FGM cylinder at its outer Ro, indicating the thermal expansion coefficient, thermal conductivity coefficient, and specific heat capacity, respectively. Therefore, Based on Eq. (1), the variation of thermal and mechanical properties of a thick-walled FGM cylinder in the direction of radius can be illustrated in Fig. (3), respectively. Additionally, Table (1) shows the thermo-mechanical characteristics of SIC and TZM under study.

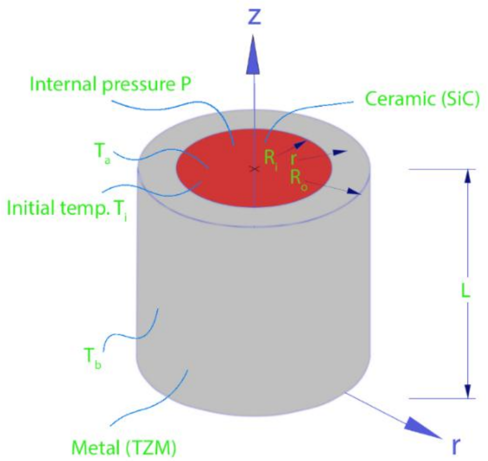

Figure 2: Configuration of model geometry, loads, and boundary condition

2.2. Loading and boundary conditions

The boundary conditions considered for the FGM cylinder involve a time-dependent mechanical load represented by internal pressure (P) and a transient thermal load manifested as temperature distribution (T). The temperature distribution arises due to transient heat conduction across the cylinder thickness, resulting from temperature differences at the inner and outer surfaces of the cylinder. Mathematically, these boundary conditions can be expressed as follows:

- Mechanical boundary conditions:

$$\sigma_r = -P \quad when \quad r = R_i$$
$$\sigma_r = 0 \quad when \quad r = R_o$$

Where, σr denotes the radial stress as a function of both radius r and time t. While, the internal time-dependent pressure P, assumed to follow Eq. (2), is depicted in Fig. (4) and identified as the mechanical boundary condition in COMSOL.

$$P = p_0\,(1 + exp\hat{}(-nt)) \qquad (2)$$

In the Eq. (2), p_0 represents the initial value or the starting point of the function P, such that when t = 0, P=2 p_0. The parameter n referred to as the decay rate constant, determines how quickly the term exp^(-nt) decays as t

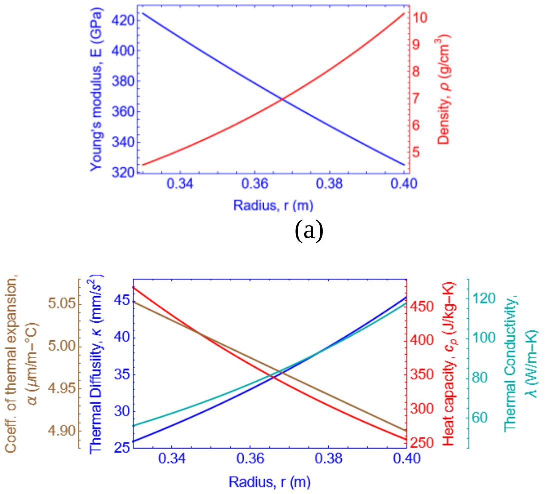

Figure 3: TZM-SiC FGM cylinder material properties with respect to cylinder radius (a) mechanical, (b) Thermal

increases, in our case for the purpose of this analysis, the initial value p_0 and the decay rate constant n are arbitrarily assumed to equal 10MPa and 0.05, respectively.

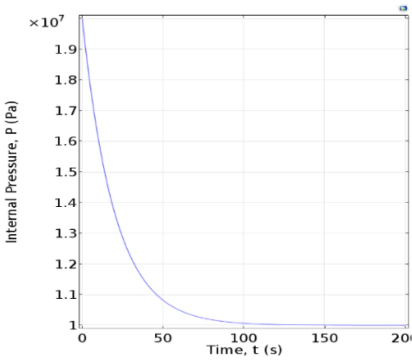

Figure ٤: Variant pressure applied to the internal surface of the TZM-SiC FGM cylinder over time, as determined using COMSOL

Table 1. TZM-SiC FGM cylinder material properties with respect to cylinder radius (a) mechanical, (b) Thermal

Material	Young's modulus, E [GPa]	Thermal expansion coeff., α [×10^{-6} 1/K]	Thermal conductive coeff., k [W/(mK)]	Density, ρ [Kg/m3)]	Poisson's ratio, υ	Specific heat, cp [J/KgK]	Thermal Diffusivity, κ [×10^{-6} m^2/s]
SIC	476	5.12	41	3210	0.29	627.6	20.35
TZM	325	4.9	118	10160	0.29	255	45.54

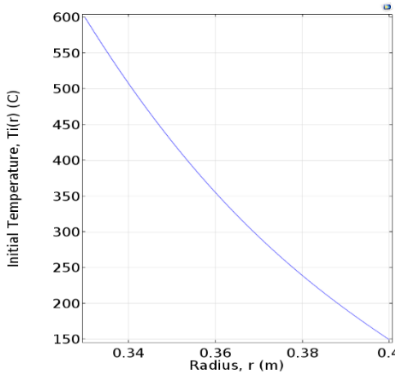

Figure ٥: Initial Temperature across the thickness of the TZM-SiC FGM cylinder in relation to its radius

- Thermal boundary conditions:

$$T = T_a \quad on \quad r = R_i$$
$$T = T_b \quad on \quad r = R_o$$
$$T_i \quad on \quad R_i \geq r \geq R_o$$

Where, as depicted in Fig. (2), the initial temperature Ti, described by Eq. (3) as a function of r acting on the cylinder's inner surface, along with prescribed surface temperatures T_a=120 ℃ and T_b=75 ℃ for the internal and external surfaces, respectively, which are selected arbitrarily for the purpose of analysis, are considered as the thermal boundary conditions and identified as such in COMSOL.

$$T_i(r) = \frac{\left(Ei[\gamma]-Ei\left[\frac{r\gamma}{R_O}\right]\right)T_i+\left(Ei\left[\frac{r\gamma}{R_O}\right]-Ei\left[\frac{R_i\gamma}{R_O}\right]\right)T_o}{\left(Ei[\gamma]-Ei\left[\frac{R_i\gamma}{R_O}\right]\right)} \qquad (3)$$

Where the temperatures denoted as Ti and To represents the inner surface temperature and the outer surface at the steady state phase and are assumed for the study as Ti=600 ºc, To=150 ºc. Additionally, Ei denotes the exponential integral function.

2.3. Mesh generation

It's crucial to emphasize the high impact of the meshing on the accuracy of numerical simulation results. Therefore, the Quadrilateral element type is selected for meshing due to its suitability for the geometry, considering its advantages; maintain uniformity, easier to visualize, computational efficiency.

For meshing convergence process of the model, careful consideration is given to the mesh size and refinement in the radial direction to achieve accurate and reliable results. The convergence study involves selecting different parameters for meshing such as maximum element size (max_e), and minimum element size (min_e) in addition to refining meshing in the radial direction by mapping the meshing through specifying the required number of elements in the radial direction (N-e) and recording the total number of elements generated for each set of parameters. Several mesh sizes are systematically chosen varying from very coarse mesh size to extremely fine mesh size. Table (2) displays the different parameters selected for the meshing selection study, along with the total number of elements corresponding to each utilized set of parameters. Additionally, Fig. (6) illustrates the refined geometry meshing as per parameters set number (S_n)=6.

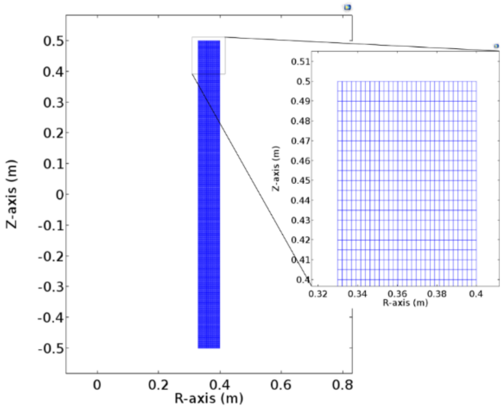

Figure ٦: COMSOL Axisymmetric geometry configuration of cylinder model (2D longitudinal section representing the cylinder wall section) with meshing refined considering parameters set number (S_n)=6.

Convergence testing is conducted through running simulations with the different sets of mesh densities, ranging from parameter mesh set S_n=1 to S_n=8, and comparing the results. A convergence plot is generated using the Hoop stress at point (r=0.36m, z=0m) to compare and evaluate the optimum element mesh size. As illustrated in Fig. (7), there is no significant change in the results starting from parameter set S_n=6, indicating that the mesh density shown in Fig. (6) is the optimal mesh density where further mesh refinement does not lead to significant improvements in solution accuracy.

2.4. Solver Selection

For the COMSOL study settings, considering both mechanical and thermal loads, a solid mechanics interface is chosen as the Structural Mechanics Physics to express and solve the mechanical part. Additionally, a heat transfer in solids interface is selected to express and solve the thermal part. The solver setting is configured as time-dependent, as both the mechanical and thermal loadings vary with time.

2.5. Numerical Simulation

The simulations are executed to solve the defined

Table 2. The selected different parameters utilized in the meshing selection study.

Parameters set number (S_n)	1	2	3	4	5	6	7	8
Maximum element size (max_e), m	0.67	50	0.25	0.25	0.05	0.01	0.01	0.005
Minimum element size (min_e), m	0.002	3E-04	3E-04	1.25E-04	7.5E-05	2E-05	2E-05	2E-05

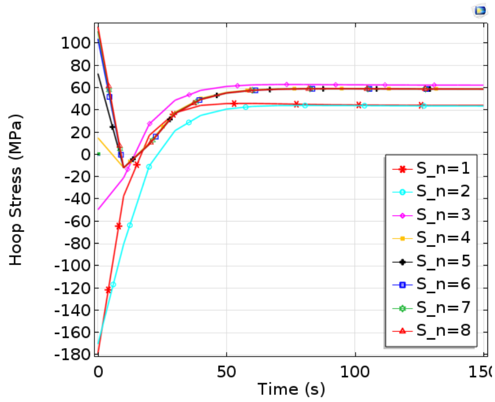

Figure 7: Convergence plot for mesh convergence study through in vestigating the hoop stress values at point (r=0.36m, z=0m) and considering meshing parameter sets (S_n=1 to S_n=8).

problem using the selected mesh and solver settings. The numerical model incorporates the specified boundary conditions, material properties, and loading scenarios to simulate the thermo-mechanical behavior of the metal-ceramic functionally graded material (FGM) thick cylinder under transient thermal loads and internal pressure. By systematically stepping through the time domain and accounting for the dynamic evolution of temperature and stress fields, the simulations aim to provide insight into the complex interactions within the FGM structure.

3.RESULTS AND DISCUSSION

3.1. Temperature distribution

The thermo-mechanical coupling is enabled, the solver settings are configured for time dependent and the time is set up for stepping from 0 to 400 seconds, using a time step of 10 seconds. By applying the transient heat transfer analysis, the temperature distribution profile through the cylinder wall thickness at z=0—where the temperature variation occurs only in the radial direction—is obtained. As the results tend to remain constant or steady after 150s, the temperature distribution is presented at four arbitrary times (t=30s, 50s, 80s, and 150s) to illustrate the variation of results through the cylinder thickness within the time span up to 150s as shown in Fig. (8).

As shown in Fig. (8), the temperature at the inner and outer surfaces remains constant at the prescribed values. However, due to the initially higher temperature distribution illustrated in Fig. (5) and the transient conduction, temperatures throughout the thickness, except at the inner and outer radii, decrease over time until reaching a steady state. This observed trend aligns with the temperature distribution depicted in the referenced study [35].

3.2. Stress distribution

The numerical stresses' results are obtained and plotted. These results indicate that radial and axial stress values increase with r and decreases over time, as shown in Fig. (9)(a) and (c), respectively. Meanwhile, the hoop stress Fig. (9)(b) exhibits maximum values at the inner and outer radii, decreasing towards the middle of the wall thickness for time up to 50s. For values exceeding 50 s, the hoop stress increases in the radial direction, with minimum values at the inner radius. Additionally, the von Mises stress values Fig. (9)(d) are decreases with r and decreases over time. The forementioned stresses variations within the cylinder thickness over time aligns with the variations in internal pressure Fig. (4) and temperature distribution Fig. (8) over the FGM cylinder.

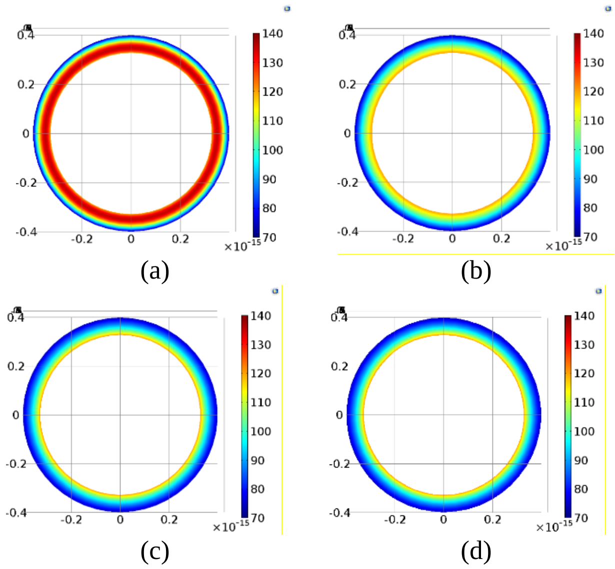

Figure 8: The temperature distribution profile in Celsius degrees through the cylinder wall thickness at z=0 presented at different times (a)t=30s, (b)50s, (c)80s, and (d)150s.

3.2.1. Analysis of Stress Distribution Patterns

To further elucidate the significance of the findings, a detailed analysis of the stress distribution patterns observed in the functionally graded material (FGM) cylinder is provided. The implications of these patterns on material performance, particularly in comparison to homogeneous and heterogeneous materials, are discussed as follows:

- The radial stress (Fig. (9)(a)) shows a decreasing trend from the inner to the outer radius, with initially high compressive values at the inner radius that reduce over time. This behavior is influenced by the gradation in material properties of the FGM, which effectively distributes the internal pressure more evenly across the thickness. The decreasing stress over time aligns with the reduction in internal pressure (Fig. (4)), indicating that FGMs can effectively mitigate high-stress concentrations over the duration of the transient load. This property is particularly advantageous in applications where the internal pressure

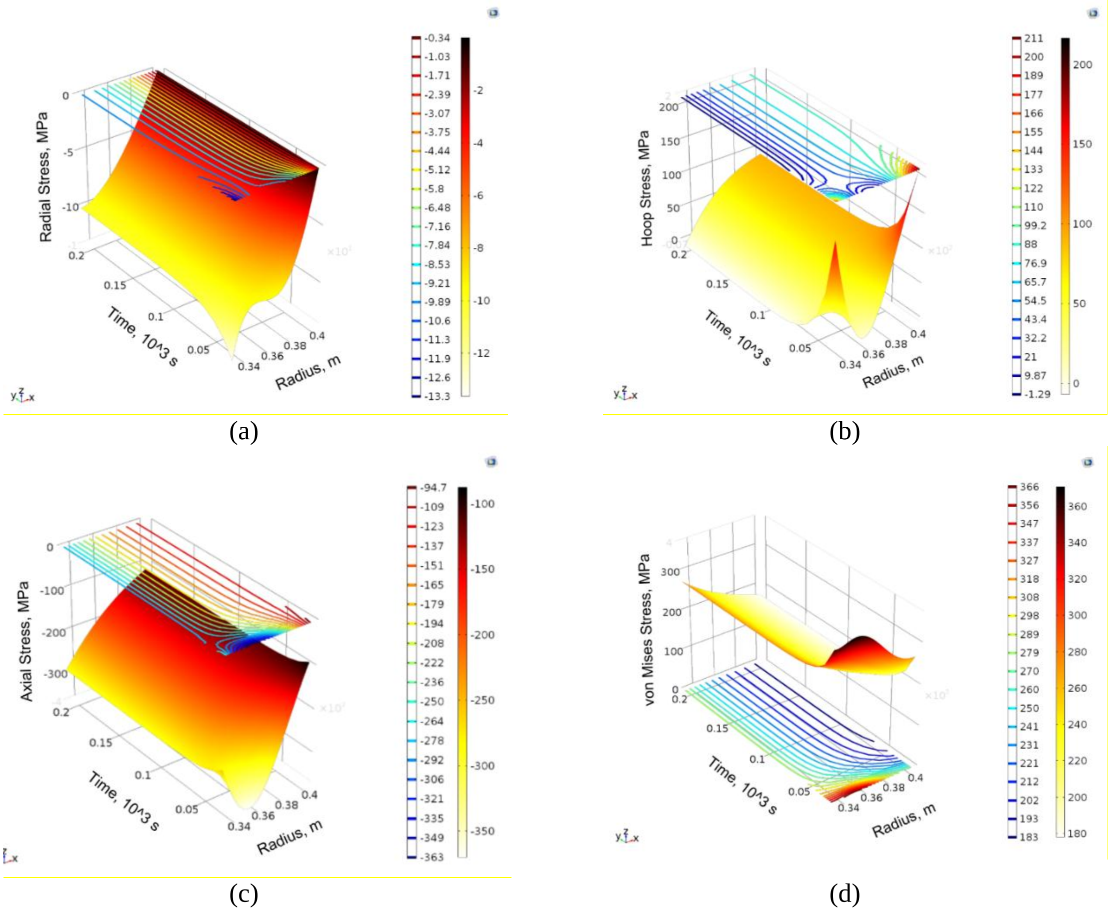

Figure 9: 3D and contour plots for (a) radial, (b) hoop, (c) axial, and (d) von Mises stress, with respect to radius (r in meters) represented on the x-axis and time (t in 10^3 seconds) represented on the y-axis.

varies, as FGMs can better absorb and distribute these changes, reducing the risk of structural failure compared to homogeneous materials.

- The hoop stress (Fig. (9)(b)) initially exhibits maximum values at both the inner and outer surfaces, decreasing towards the mid-thickness. Over time, the stress stabilizes and the maximum values shift towards the outer radius. This trend corresponds with the temperature distribution (Fig. (8)), where thermal gradients are initially high but become more uniform over time. The ability of FGMs to have varying material properties through the thickness allows for better accommodation of these thermal gradients and mechanical loads, reducing the likelihood of thermal-induced stress concentrations. This is particularly beneficial for components subjected to cyclic thermal loads, as FGMs can adapt to changing conditions more effectively, reducing thermal fatigue.
- The axial stress (Fig. (9)(c)) decreases from the inner to the outer radius and reduces over time. The axial stress shows maximum negative values at the inner radius initially, which then diminish towards the outer radius and over time. This behavior is directly related to the reduction in both mechanical load (Fig. (4)) and thermal load (Fig. (8)) over time. The gradation in the FGM provides a smoother stress distribution, reducing the likelihood of buckling or axial deformation. This is advantageous over homogeneous materials, which might not handle axial loads as effectively under similar conditions, leading to better structural integrity and stability in the long term.
- The von Mises stress (Fig. (9)(d)), representing the equivalent stress combining radial, hoop, and axial stresses, is highest at the inner radius initially but decreases over time due to the decay of loading conditions. This pattern shows that the trends in von Mises stress align with those of the radial, hoop, and axial stresses, confirming the consistency of the stress distribution results. The gradation of the FGM in the radial direction effectively reduces high-stress concentrations by gradually varying the material properties. The reduction in von Mises stress over time is crucial for ensuring the component does not reach yielding conditions prematurely. This implies a longer operational lifespan and greater reliability of the FGM cylinder under varying load conditions compared to traditional homogeneous materials.

3.2.2. Influence of Material Properties Gradient

It's important to emphasize that the radial gradient of the FGM properties, with enhanced thermal properties at the surface subjected to the highest thermal loads as the thermal loads have the great impact on the results [35], significantly impacts the stress distribution. These tailored property distributions, as highlighted by Habib et al. [9], reduce stresses on surfaces exposed to intense thermal loads, ultimately improving stress distribution and lowering overall stress levels compared to pure materials.

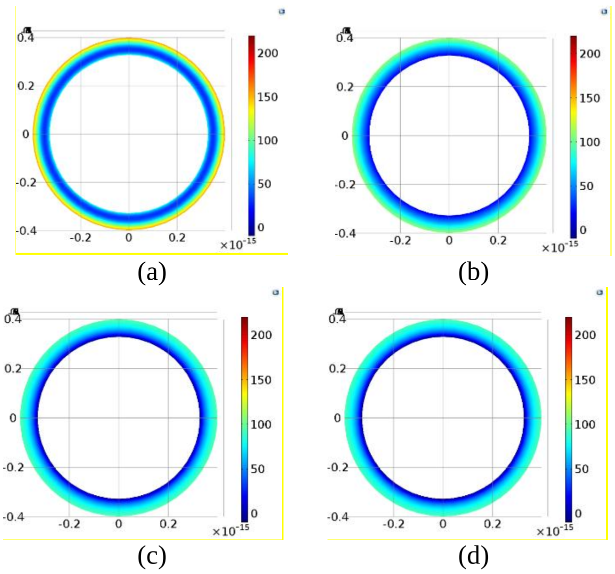

Figure 10: The hoop stress (MPa) distribution through the cylinder wall thickness at z=0 presented at different times (a)t=30s, (b)50s, (c)80s, and (d)150s.

3.2.3. Cross-Sectional Analysis

To illuminate the changes in stress distribution through the thickness over time, cross sections illustrating the variation of stresses across the FGM cylinder wall thickness at z=0 m and at different times (t=30 s, 50 s, 80 s, and 150 s) are examined. Significant variations are observed in all results up to 50 s. Subsequently, the changes become less pronounced, with smaller variations persisting until reaching a steady state phase at approximately 150 s. These trends are depicted in Fig. (10) for hoop stress, Fig. (11) for radial stress, Fig. (12) for axial stress, and Fig. (13) for von Mises stress, respectively.

By analyzing these stress distributions at various time points, a comprehensive understanding of the dynamic response of the FGM cylinder to mechanical and thermal loads is obtained. This knowledge is crucial for optimizing the design and ensuring the structural reliability and safety of the component under operating conditions. Furthermore, the insights gained from these stress distributions can inform future design iterations and improvements to enhance the performance and longevity of FGM-based structures in real-world applications.

3.2.4. Comparison with Analytical Solutions

The comparison of stress representations between the present numerical solution and results obtained from the analytical solution [35] reveals generally good agreement across the majority of the plot area, with minor deviations observed, particularly in the region near time zero. This discrepancy appears to stem from convergence issues in both the analytical and numerical solutions. In the analytical solution, the use of Taylor approximation to calculate f_2, f_3, and f_{15} [35] introduces truncation errors due to the neglect of high-order terms [36], affecting calculation accuracy. On the other hand, in the numerical solution, the maximum error is evident near the inner cylinder surface and close to time zero, indicating a potential discretization error.

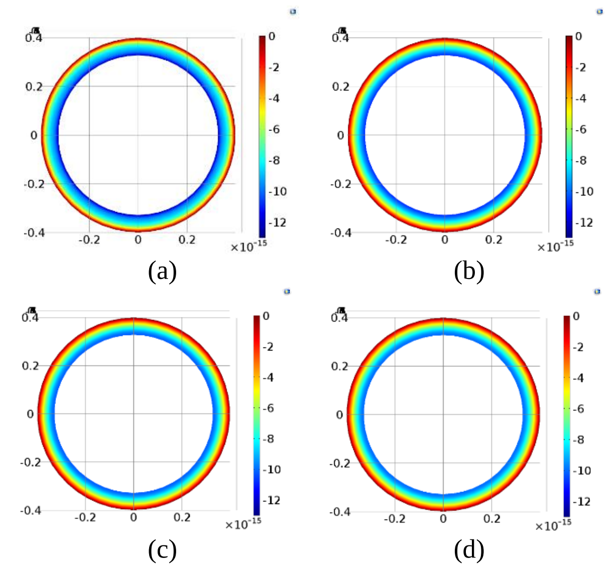

Figure 11: The radial stress (MPa) distribution through the cylinder wall thickness at z=0 presented at different times (a)t=30s, (b)50s, (c)80s, and (d)150s.

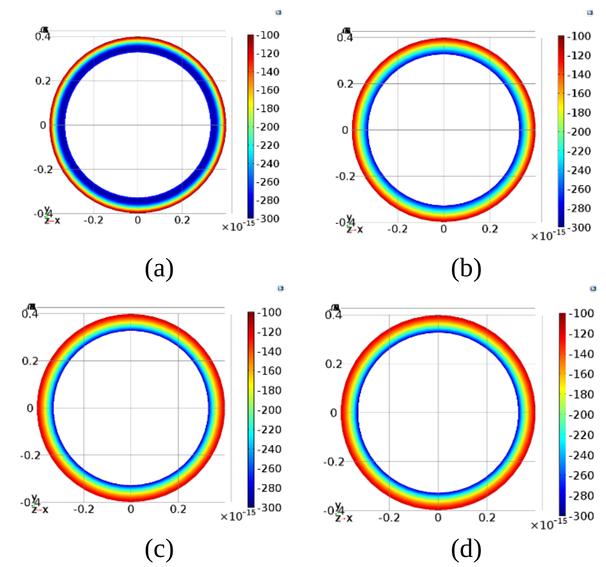

Figure 12: The axial stress (MPa) distribution through the cylinder wall thickness at z=0 presented at different times (a)t=30s, (b)50s, (c)80s, and (d)150s.

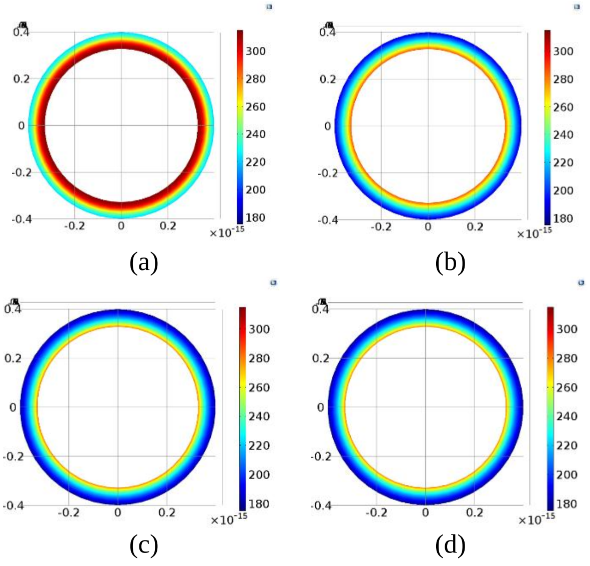

Figure 13: The von Mises stress (MPa) distribution through the cylinder wall thickness at z=0 presented at different times (a)t=30s, (b)50s, (c)80s, and (d)150s.

3.2.5. Validation of Results

To further evaluate the agreement between the numerical and analytical solutions, the variation of hoop stress with respect to time at radii of 350mm and 370mm is plotted for both solutions, as shown in Fig. (14) and Fig. (15) respectively. It can be determined that the results from the analytical solution [35] are verified. The maximum errors between these two curves at radii of 350mm and 370mm are approximately %7.6 and %9.8, respectively. This analysis underscores the importance of utilizing numerical approaches to validate analytical results and to gain a comprehensive understanding of the limitations in capturing the complex behavior of FGM cylinders under mechanical and thermal transient loading conditions.

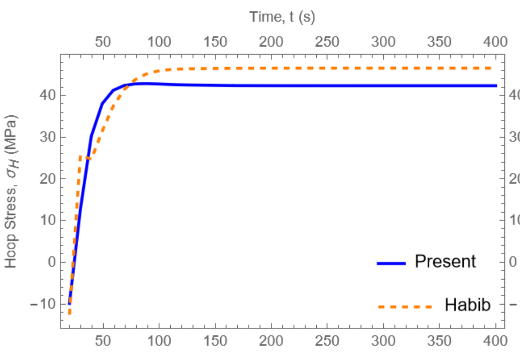

Figure 14: The variation of hoop stress with respect to time at radius 350mm for both numerical and analytical solution.

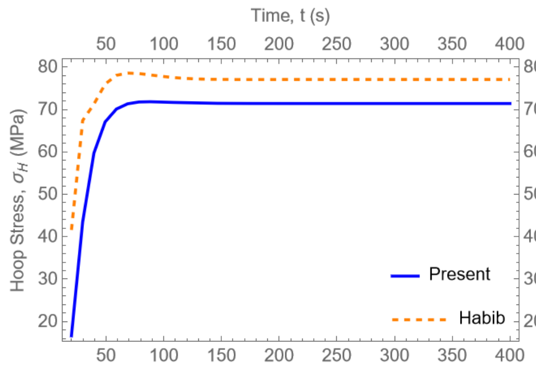

Figure 15: The variation of hoop stress with respect to time at radius 370mm for both numerical and analytical solution.

4.CONCLUSION

In this investigation, a numerical analysis using COMSOL Multiphysics validates the analytical solution proposed by [35] for a metal-ceramic FGM thick cylinder subjected to transient thermal loading and internal time-dependent pressure. The study confirms the numerical approach's alignment with analytical predictions, providing a reliable tool for understanding FGM behavior under such conditions. The findings demonstrate significant agreement between numerical and analytical results, affirming the methodology's efficacy. In future research, exploring complex geometrical models, incorporating static or time-dependent external loads, and performing experimental validation of those numerical findings is essential to enhance understanding of FGM performance in real-world conditions and drive further advancements in this field.

5.LIMITATIONS OF THE STUDY

5.1. Assumptions Made and Impact on Results:

- Constant Poisson Ratio: The Poisson ratio is considered constant to align with the analytical solutions used for validation. The impact on the results is that this assumption provides a simplified yet insightful understanding of the behavior of metal-ceramic FGM thick cylinders under transient thermal and mechanical loads. However, it might introduce deviations from the real material behavior.
- Temperature-Independent Thermo-Mechanical Properties: Assuming the thermo-mechanical properties are temperature independent for the same previous reason and dismissing the dependent characteristic of the thermo-mechanical properties on temperature. This simplification makes the analysis more manageable but may not fully capture the material behavior under varying temperatures,

potentially leading to deviations from real-world performance.

5.2. Experimental validation

- The numerical results were validated against established analytical solutions, ensuring a preliminary level of accuracy. However, the lack of experimental validation is acknowledged as a limitation that needs to be addressed in future work.

6.FUTURE WORK:

- Future research shall obtain empirical data for the specific FGM materials studied. This will help validate the numerical results and refine the assumptions made.
- Future studies will incorporate more complex geometrical models to better simulate real-world applications and capture the nuances that simpler models might miss.
- Including static or time-dependent external loads in the study will provide a more comprehensive analysis of the FGM thick cylinder's behavior under different operational conditions.

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