An Experimental and Numerical Investigation for Characteristics of Submerged Hydraulic Jump over Corrugated Beds

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ABSTRACT
The present study was executed to compare flow parameters of submerged jump under gate using physical and numerical models. Twenty experimental runs were carried out to study the impact of spaced corrugated beds on basic characteristics of submerged hydraulic jump for values of Froude numbers ranging from 1.5 to 9.5. Experiments were conducted also for smooth bed. Submerged jump on best spaced corrugated bed depending on experimental runs was simulated by using VOF method in FLUNET Software and k-ε and RNG k-ε turbulent models are used. To simulate a highly transient and turbulent flow conditions a free surface CFD numerical model has been applied. The results revealed that there is a great coincidence between experimental and numerical simulation, corrugated beds reduced jump length and sequent depth by average 5.69% and 12.96% respectively, k-ε and RNG k-ε turbulence models have the ability to simulate this phenomenon and finally the RNG k-ε model give results more consistent with laboratory experiments.

Keywords: Submerged Jump, Froude Number, Spaced Corrugated Beds, Volume of Fluid (VOF), FLUNET and Computational Fluid Dynamic (CFD)

1- INTRODUCTION
Hydraulic jump is a process of transformation of a supercritical flow to a subcritical flow by distracting a great amount of energy. A hydraulic jump is occurred when the flow suddenly changes from supercritical flow (minimum depth accompanying by high velocity) to subcritical flow (maximum depth accompanying by low velocity), Chow [1]. It occurs in an open channel flow under sluice gates, over the top of spillways /weirs or when a steep slope of bed channel suddenly turns to flat (horizontal). The applications of hydraulic jump in flow through open channel are vital as energy distracting device through different hydraulic structures, purification of water by mixing it with chemical additives, and aeration of flows. Many of laboratory tests and field studies have been carried out to define the characteristics of hydraulic jump phenomena such as its location, the sequent depths, its length, the energy distracting, and its efficiency. Hydraulic jump on smooth bed has been investigated experimentally by many researchers Vischer and Hager [2]. Recently, some studies have been carried out on corrugated beds. Hydraulic jump was first investigated experimentally by Bidone [3]; thereafter, many studies were made and the results were quoted by many engineers. Rajaratnam [4] performed the first methodical studies on the hydraulic jumps over corrugated bed. He proved that the roller length \( L_r \) and the jump length \( L_j \) upon rough bed would decrease significantly in comparison to the smooth bed. Negm et al. [5] used two types of roughness element; they found that 13% and 16% roughness intensities provide the minimum relative jump length when hexagonal and cylindrical roughness elements were used for roughening the bed respectively. Hughes and Flack [6] performed experimental researches on hydraulic jumps on corrugated bed. They concluded that roughness of boundary layer will absolutely reduce both the jump length and subcritical depth depending on the Froude number and relative bed roughness.

Ead and Rajaratnam [7] carried out an experimental investigation on hydraulic jumps upon round shape corrugated bed. Froude numbers are ranged from 4 to 10 and the values of the relative roughness were ranged between 0.25 and 0.5. They noted that the corrugated beds were the reason for reducing the required tailwater depth of that which occurs over smooth beds and also the length of the jump is approximately equal to half of that which occurs over smooth bed.

El-Azizy [8] investigated the impact of various intensities of bed roughness on submerged hydraulic jump. Tokyay et al. [9] carried out many laboratory experiments to define the impacts of prismatic shape roughness elements on characteristics of the jump. They found that using prismatic roughness elements significantly reduced the jump length and jump sequent depth and induced more energy dissipation compared to a classical jump. Pagliara et al. [10] investigated the factors that affect both sequent depth and jump length over homogenous and non-homogenous rough bed channels downstream of block ramps. Ezzizah et al. [11] conducted experimental works using new different

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shapes of roughness like U-shape, cubic shape and curved shape. They conclude that U-shape improved characteristics of hydraulic jump from other shapes. Deshpande M. M. and Talegonkar S.D. [12] studied hydraulic jump properties considering different channel bed condition (different roughened shapes with different dimensions and spacing) and the results confirmed that corrugated bed increases bed shear stress so it always shows best behaviour than a smooth bed for decreasing both depth and length of hydraulic jump.

Shaker A. J. et al. [13] studied experimentally the effect of prismatic sill under gate on the characteristics of hydraulic jump by changing the sill height several times and compare with that without sill. The results explained that the performance of free flow increases up to 25% in case of using prismatic sills under sluice gate of that which occurs without sill. Numerical methods for studying of hydraulic jump on corrugated beds are complicated process because of boundary layer problem, complications of turbulent flow and diffusion process contributed between bed, walls and rotating water surface.

Turbulence models are used to simulate hydraulic jump is a 2-phase and accurate results can be obtained by applying VOF (volume-of-fluid) method. The standard $k-\varepsilon$ and a multi scale $k-l$ models were applied to simulate turbulent flow of hydraulic jump by Zhao Q. and Misra S. K. [14].

The precursory results confirmed that the numerical model corresponds well with the experimentally measurements for the turbulent flow. Computational fluid dynamics (CFD) program was used by Savage B. M. and Johnson M. C. [15] to simulate the physical model which was conducted to investigate flow over ogee spillway.

The results explained that for free flow over an ogee spillway; numerical models are considered as an advanced tools to determine flux and pressure on spillway and achieved good agreement between results of numerical and physical models. Ramazza M. et al. [16] used the near-critical flow of the undular jump to validate the numerical model.

The numerical simulations are performed by using Fluent with VOF multiphase model; pressure based explicit solver, geometric interface re-construction and standard $k-\varepsilon$ turbulence model. Abbaspour A. et al. [17] evaluated the jump on a corrugated bed by applying 2-dimensional numerical simulation and using both standard $k-\varepsilon$ and RNG $k-\varepsilon$ models as shown in Fig. 1.

Figure 1 Simulation of free water surface by using VOF method.

They concluded that the free water surface can be simulated by applying VOF method and the results confirmed that both VOF method and $k-\varepsilon$ turbulent model are considered as suitable methods to predict free water surface profile under sluice gate. Standard $k-\varepsilon$ model was used successfully by Ebrahimi S. et al. [18] to simulate hydraulic jump on triangular and rectangular rough beds and the results confirmed that numerical scheme was used to predict all characteristics of free turbulent water surface with an accuracy range reaches to 4.4%.

Ebrahim N. [19] used FLUENT software program with three turbulent models (RNG $k-\varepsilon$, standard $k-\varepsilon$, and Realizable $k-\varepsilon$) to simulate the flow through fish-way that is constructed beside the dams and compare the results with instrumentation data. The results showed that $k-\varepsilon$ standard model showed more efficiency in flow numerical modeling in the weir and orifice fish-way because of modelling the effect of spin than other models and similarity of answers to the laboratory models. The results also indicated that with respect to the velocity vector, the middle part of the pool and near to the middle wall the upstream is the safe area for fish aggregation and thus it is optimal economically to transfer fish. Ammar G. and Kouro N. [20] used FLUENT software to simulate the hydraulic jump index for type I stilling basin which included investigating the impact of Froude number, spillway length and width on the hydraulic characteristics and the water surface profile. The results confirmed that FLUENT software is one of the most powerful soft-wares in this field.

From the author’s point of view, the studies which were conducted on submerged hydraulic jump are too few but rather rare especially on corrugated beds so this research may be a good starting point for the study by applying an experimental and numerically investigation.
The objective of this research is to investigate submerged jump on rough beds by experimental works using spaced strip triangular sheets and compare the best results of runs with results of FLUENT software program using two turbulent models (standard $k-\varepsilon$ turbulence and RNG $k-\varepsilon$). Finally, 2-Dimension Reynolds-averaged Navier Stokes equations are solved by the finite volume method which is considered as the basic method of CFD program.

2- EXPERIMENTAL PROCEDURE

The experiments have been conducted in the laboratory hydraulic flume of the Hydraulics Research Institute (HRI) of National Water Research Centre (NWRC). The flume is established of bricks stamped with smooth cement mortar except Plexiglas part for sides with length of 2.25 m to facilitate the observation process. Submerged hydraulic jump was created in a rectangular flume which its dimensions are 0.75 m width, 0.70 m depth and 25 m length. The dimensions of cross section of the triangular corrugated sheet beds are 0.04 m base and 0.04 m altitude. Aluminium triangular sheets corrugated beds were set up on the bed of flume in which that the crest of corrugated bed sheets laid at the same level of upstream bed as shown in Fig. 2.

Experiments runs have been conducted for three values of the distance $(S) = 4$cm, 12cm and 20cm as shown in figures 3, 4 and 5. The distance between each two consecutive crests of corrugated bed sheets $(S)$ and its height $(t)$ are shown in Fig 6. An experiments for smooth bed were conducted also to use its results as a reference to the cases of corrugated bed using Froude number $F_r = 1.5, 2.75, 4.5, 6.25$ and $9.5$ for each value of distance $(S)$. By that all types of submerged hydraulic jump were studied.

Figure 2 General side view of experimental flume.
3. NUMERICAL MODEL

Incompressible two-phase flow is governed by conservation of mass and momentum equations, known both water and air phases. These laws are explained through the Navier–Stokes (N-S) equations, which, in their original form, involved all known external and internal effects of the fluid motion. The governing equations that control the physics of fluid mechanics which are play an active role in research of computational fluid dynamics (CFD) are continuity equations, Navier-Stokes equations and energy equations. The hydraulic jump has been investigated using mathematical models based on the Navier–Stokes equations, free surface tracking methods like the Volume Of Fluid (VOF) method Hirt, C.W. and Nichols, B.D. [21].

In present research submerged hydraulic jump on corrugated beds sheets are numerically studied on the best results of experimental works for different Froude numbers using both the standard $k-\varepsilon$ and renormalization group RNG $k-\varepsilon$ turbulent models and two-phase flow theory.

### 3.1. Governing equations of (CFD)

The key governing equations are the unsteady incompressible flow for two-dimensional continuity equation and Reynolds- Navier-Stokes equation for both air and liquid phases Liu et al. [22].

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_j) = 0
\]

(1)

\[
\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_j}(\rho U_i U_j) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i}(\mu + \mu_t) \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \rho g_j
\]

(2)

\[
\rho = \alpha_A \rho_A + \alpha_w \rho_w
\]

(3)

\[
\mu = \alpha_A \mu_A + \alpha_w \mu_w
\]

(4)

Where:

- $U_i$ and $U_j$: the velocity components,
- $\alpha_A$ and $\alpha_w$: the volume of fraction of air and water phases respectively,
- $\rho_A$, $\rho_w$ and $\rho$: the density of air, water and mixture of them respectively,
- $\mu_A$, $\mu_w$, $\mu$ and $\mu_t$: the viscosity of air, water, mixture and the vortex viscosity, respectively
- $g$: the gravity acceleration

### 3.2. Volume of fluid (VOF)

The volume-of-fluid (VOF) technique can skillfully deals with as an interface for both free surface and the fluid. VOF method is supported on the determination of a mission of volume of fraction, whose its value at any grid point is equal to one unit if filled with the fluid while a zero value if the cell contains no fluid as shown in Fig. 7.

Essentially the Geometric Reconstruction scheme was applied in this study. The interface between fluids was introduced by applying a piecewise linear approach through this method which is most accurate method and also it is usable for general unstructured meshes.
The VOF scheme uses a function $F(x, y, t)$ to specify the free surface. The function $F$ is calculated from this equation:

$$\frac{\partial F}{\partial t} + \frac{\partial F U_j}{\partial x_i} = 0$$

(5)

Where:

$F = A$ unit value of cell full of fluid which ranged from 0 to 1

3-3- Standard $k - \varepsilon$ model

Standard $k - \varepsilon$ turbulence model was used to simulate the submerged hydraulic jump. It is given by the following equations:

$$\rho U_i \frac{\partial k}{\partial x_i} = \mu_t \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) \frac{\partial U_j}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_i} \right) - \rho \varepsilon$$

(6)

$$\rho U_i \frac{\partial \varepsilon}{\partial x_i} = C_{1\varepsilon} \left( \frac{\varepsilon}{k} \right) \mu_t \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) \frac{\partial U_j}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) -$$

$$C_{2\varepsilon} \rho \left( \frac{\varepsilon}{k} \right)$$

(7)

Where:

$k$ and $\varepsilon$ = The kinetic energy and dissipation rate

$\sigma_k$ and $\sigma_\varepsilon$ = constants of turbulent Prandtl numbers for $k$ and $\varepsilon$ equals 1 and 1.3 respectively.

$C_1$ and $C_2$ = constants equals 1.44 and 1.92

3.4 Renormalization group RNG $k - \varepsilon$ model

Standard $k-\varepsilon$ turbulence model was used to model the hydraulic jump in the present study. It is given by this equation:

$$\rho U_i \frac{\partial k}{\partial x_i} = \mu_t \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) \frac{\partial U_j}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_i} \right) - \rho \varepsilon$$

(8)

$$\rho U_i \frac{\partial \varepsilon}{\partial x_i} = C_{1\varepsilon} \left( \frac{\varepsilon}{k} \right) \mu_t \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) \frac{\partial U_j}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) -$$

$$C_{2\varepsilon} \rho \left( \frac{\varepsilon}{k} \right)$$

(9)

Where:

$C_{2\varepsilon} = 1.68$, $\eta_0 = 4.38$, and $\beta = 0.012$ are model constants.

3.5 Model geometry and boundary conditions

The model geometry and the unstructured grids were created to analyze the hydraulic jump on a rough bed. The grids size of the model geometry = 5mm. The elementary regions and boundary conditions in the jump on corrugated beds were shown in Fig. 8.

4- RESULTS AND DISCUSSION

In the first phase of the analyses, reliability of FLUENT code is assessed. As mentioned before, the experiments are also carried out on a smooth bed as a reference case.

In the second phase, the impact of strip roughness sheets on the parameters of a hydraulic jump is investigated. Characteristics of submerged hydraulic jump on corrugated bed such as sequent depth ratio, jump length and submerged depth ratio were measured by experimental works as shown in Fig. 10 and applying standard and RNG turbulent models as a numerical analysis.
4.1- Sequent depth ratio

The sequent depth of jump \( y_2 \) can be obtained by measuring water surface profiles as shown in Fig. 10 for two turbulent models at roughness ratios \( S/t = 3 \) only whereas, it had been measured experimentally for all roughness ratios in addition to smooth bed. Fig. 11 shows relation between the sequent depth ratios \( y_2/y_1 \) of hydraulic jump versus Froude number \( F_r \) for experimental and numerical runs where \( y_1 = \) open gate. This figure shows that, the sequent depth ratios of submerged jump over all different spaced of corrugated beds is less than that over smooth bed. The results indicate that, the minimum values of sequent depth occurred at roughness ratio (\( S/t = 3 \)) and experimentally it decreases by 5.69%, 7.69%, 9.07%, 10.6% and 12.96% in comparison with that of the smooth bed for \( F_r = 1.5, 2.75, 4.5, 6.25 \) and 9.5 respectively. Also, Fig. 11 shows that, for \( S/t = 3 \) a good agreement was found between standard \( k-\varepsilon \) and RNG models with experimental results.

As can be seen in Fig. 11, at \( S/t = 3 \) the results obtained from the turbulence RNG model is closer to the experimental results. Furthermore, the convergence of solutions of RNG turbulence model was shorter than the standard \( k-\varepsilon \) model. The mean error of the sequent depth ratio of hydraulic jump between RNG turbulence model and experimental results is about 1.64%-9.61%.

The best fit lines of Standard \( k-\varepsilon \) model, RNG model and experimental at \( S/t = 3 \) and the corresponding coefficients of determination are:

\[
\left( \frac{y_2}{y_1} \right)_{\text{Standard}} = 1.08F_r^{1.007} \quad (R^2 = 99\%) \quad (10)
\]

\[
\left( \frac{y_2}{y_1} \right)_{\text{RNG}} = 1.11F_r^{1.005} \quad (R^2 = 99\%) \quad (11)
\]

\[
\left( \frac{y_2}{y_1} \right)_{\text{Exp.}} = 1.14F_r^{1.008} \quad (R^2 = 99\%) \quad (12)
\]

4.2- Submerged depth

Fig. 10 shows that, the submergence depth of jump \( y_3 \) over strip corrugated bed sheets is small than that over smooth bed. It can be noted that, there is a good agreement between experimental data with both standard \( k-\varepsilon \) and RNG \( k-\varepsilon \) turbulence models at optimal roughness ratio (\( S/t = 3 \)).

Regarding Fig. 12 demonstrates that the relative submerged ratio at optimal spacing ratio decreases by 18.75%, 29.74%, 30.73%, 40.81% and 43.26% comparison with that of the smooth bed for all range of \( F_r = 1.5, 2.75, 4.5, 6.25 \) and 9.5 respectively.

The mean error of the sequent depth of submerged jump between RNG turbulence model and experimental results is about 1.04%-7.61%.

The power relationship between \( \left( \frac{y_3}{y_1} \right) \) and \( F_r \) for standard \( k-\varepsilon \), RNG \( k-\varepsilon \) models, and experimental results can be characterized as the following equations:

\[
\left( \frac{y_3}{y_1} \right)_{\text{Standard}} = 0.87F_r^{1.02} \quad (R^2 = 96\%) \quad (13)
\]

\[
\left( \frac{y_3}{y_1} \right)_{\text{RNG}} = 0.88F_r^{1.03} \quad (R^2 = 96\%) \quad (14)
\]

\[
\left( \frac{y_3}{y_1} \right)_{\text{Exp.}} = 1.03F_r^{0.956} \quad (R^2 = 96\%) \quad (15)
\]

4.3- Hydraulic jump length

As mentioned before the length of jump \( L_j \) can be measured from the water surface profiles as shown in Fig. 10. Fig. 13 shows that, the estimated values of numerical runs of two turbulent models for jump length is in good agreement with results of experimental works.

The relative jump length at optimal spacing ratio decreases about 1.1% - 41.97% comparison with that of the smooth bed for all range of \( F_r \).

The mean error of jump length between simulated by RNG turbulent model and experimental results is ranged between1.8% - 9.3%.

The power relationship between \( \left( \frac{L_j}{y_1} \right) \) and \( F_r \) for standard \( k-\varepsilon \), RNG \( k-\varepsilon \) models, and experimental results can be characterized as the following equations:

\[
\left( \frac{L_j}{y_1} \right)_{\text{Standard}} = 3.0323e^{0.3141F_r} \quad (R^2 = 96\%) \quad (16)
\]

\[
\left( \frac{L_j}{y_1} \right)_{\text{RNG}} = 3.643e^{0.302F_r} \quad (R^2 = 98\%) \quad (17)
\]

\[
\left( \frac{L_j}{y_1} \right)_{\text{Exp.}} = 4.059e^{0.2817F_r} \quad (R^2 = 99\%) \quad (18)
\]
4-4- Jump efficiency

Although momentum is conserved throughout the hydraulic jump, the energy is not. Large amount of energy is dissipated in a hydraulic jump due to vortex and secondary wave formation. The efficiency of the hydraulic jump (η) can be expressed as η = (E₂/E₁)% where E₁ and E₂ are the energies before and after the jump.

Generally, corrugated beds have a good effect to dissipate excess energy. Fig. 14 shows that, the relative energy losses at S/t = 3 decrease about 36.47% - 88.73% comparison with that of the smooth bed for all range of F₂.

The polynomial relationship between (η) and F₂ for standard k-ε, RNG k-ε models, and experimental data can be characterized as the following equations:

\[ (\eta)_{\text{Standard}} = -0.67F²₂ + 15.34F₂ - 6.99 \]  \hspace{1cm} (R² = 99%)  \hspace{1cm} (19)

\[ (\eta)_{\text{RNG}} = -0.60F²₂ + 14.80F₂ - 4.70 \]  \hspace{1cm} (R² = 99%)  \hspace{1cm} (20)

\[ (\eta)_{\text{Exp.}} = -0.58F²₂ + 14.93F₂ - 4.59 \]  \hspace{1cm} (R² = 99%)  \hspace{1cm} (21)

4-5- Bed shear stress

The bed shear stress is obtained by using the momentum equation. The resulting equation can be written as follows:

\[ \frac{\partial}{\partial x} \rho u \frac{\partial}{\partial y} v y dz + \frac{\partial}{\partial y} \rho u \frac{\partial}{\partial y} v y dz - \frac{\partial}{\partial y} \sigma_x y dz = -\tau_b \]  \hspace{1cm} (22)

Where:
\[ \rho = \text{water density}, \]
\[ P = \text{hydrostatic pressure}, \]
\[ \sigma_x = \text{Reynolds normal stress}, \]
\[ \tau_b = \text{bed shear stress}. \]

One of the main goals of corrugated sheets is to increase the bed shear stress and, the sequent water depth and the length of the jump thus is decreased. By using the momentum equation at sections just before and after the jump, the integrated shear force equation can be written as following:

\[ F_x = \int_{x_1}^{x_2} \tau_b \ dx = (P_1 - P_2) + M_1 - M_2 \]  \hspace{1cm} (23)

Where \( P_1, P_2, M_1, M_2 \) and \( x_1, x_2 \) are pressures, momentum forces and shear force per unit width Khan AA, Steffler P.M. [23].

Where:
\[ \varepsilon = \text{shear stress coefficient}, \]
\[ F_x = \text{shear force}, \]
\[ \gamma = \text{unit weight of water}, \]

In Fig. 15 the variance of shear force coefficient \( \varepsilon \) with Froude number \( F_2 \) is shown for numerical and experimental results. Figure 13 shows that the results of both experimental works and turbulent models give good evaluations of the coefficient of shear force. Figure 13 reveals that, variation in shear force coefficient is very small at low values of Froude number\( F_r \).

The polynomial relationship between \( \varepsilon \) and \( F_r \) for standard k-ε, RNG k-ε models, and experimental results can be characterized as the following equations:

\[ (\eta)_{\text{Standard}} = 0.72F_r² + 1.19F_r + 1.78 \]  \hspace{1cm} (R² = 99%)  \hspace{1cm} (24)

\[ (\eta)_{\text{RNG}} = 0.71F_r² + 1.82F_r + 1.173 \]  \hspace{1cm} (R² = 99%)  \hspace{1cm} (25)

\[ (\eta)_{\text{Exp.}} = 0.64F_r² + 2.41F_r + 0.85 \]  \hspace{1cm} (R² = 99%)  \hspace{1cm} (26)

5- Conclusions

This study suggests that triangular strip sheets are used as an alternative solution to stabilize the location of a hydraulic jump. Experimental and numerical investigations both show that when strip triangular sheets are introduced to the channel bed, they have a positive effect on the characteristics of a submerged hydraulic jump. Both standard k-ε and RNG k-ε models have the ability to simulate turbulence flow and solve different hydraulic problems with high efficiently such as hydraulic jump. The following prominent conclusions can be drawn:

- The present study confirmed the effectiveness of corrugated beds for energy dissipation at downstream hydraulic structures.
- The preliminary results show that the reliability of a CFD model, FLUENT software agrees adequately with the experimental measurements.
- Comparison of the predicted forward parameters of submerged hydraulic jump with the corresponding measured profiles confirms that both turbulence models have the ability to simulate this phenomenon; with intent on the RNG k-ε model have best agreement with the experimental results.
- Reduction of length and sequent depth of hydraulic jump mainly depended on value of Froude number. For smaller values the amount of reduction is low and vice versa.
- Corrugated bed increases bed shear stress so it always shows best performance than a smooth bed with reducing jump length and sequent depth by average 5.69% and 12.96% respectively.

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**Figure 11** Sequent depths ratio simulated using two turbulence models with experimental results.

**Figure 12** Submerged depth ratio simulated using two turbulence models with experimental results.
Figure 13 Jump length ratios simulated using two turbulence models with experimental results.

Figure 14 Efficiency simulated using two turbulence models with experimental results.
Figure 15 Shear force coefficient estimated from two turbulent models and experimental results

References


